Journal for Technology of Plasticity, Vol. 36 (2011), Number 2

DOI: 10.2478/v10211-011-0008-x

PREDICTION OF FLOW STRESS OF ALLOYED STEEL IN HOT FORMING BY APPLYING DIFFERENT MATHEMATICAL MODELS AND DESIGN OF EXPERIMENT

Velibor Marinković

University of Niš, Faculty of Mechanical Engineering, Aleksandra Medvedeva 14, Niš, Serbia

ABSTRACT

In the past decades, a great number of mathematical models have been developed to describe the rheological behaviour of metals and alloys in hot forming conditions. Many different approaches have been used for determining the constitutive equation, i. e. the complex and non-linear relationship between flow stress and various forming process factors. In this paper some relatively simple mathematical models for predicting the flow stress of alloyed steel during the hot forming process were investigated. In order to model the constitutive response of the material, the flow stress was determined as a function of main factors, such as strain, strain rate and temperature. After analyzing the results of calculation, the constitutive low was selected that represents the best-fit flow curve model. The absolute percentage errors and correlation coefficient indicate that the proposed flow curve could predict the flow stress in real forming conditions with good correlation and generalization. Also, this flow curve can easily be utilized in engineering calculations and numerical simulations.

***Key words**: flow curves, hot forming, design of experiment, alloyed steel*

1. INTRODUCTION

The knowledge of material behaviour in hot forming condition is of great importance to engineers-designers of metal forming processes. Material behaviour in forming processes depends on its rheological properties, deformation process scheme, tribological and other conditions of forming process. The development and improvement of metal forming processes are largely based on the knowledge of the behaviour of metals and alloys in complex forming conditions. It is also a fundamental prerequisite for a successful modeling and optimization of metal forming processes and a deeper study of the nature of plastic deformation.

During hot forming processes metals and alloys suffer many physical and structural changes, the most important of which are [1]: (a) strain/strain rate hardening; (b) grain growth; (c) phase transformation; (d) static/dynamic recovery and recrystallization; (e) deformation texture and anisotropy; (f) internal failure and damages, etc. Many mechanical features and properties of the workpiece material may be changed due to the abovementioned phenomena as material responses undergoing the plastic deformation: (a) resistance of plastic deformation; (b) yield stress, strength, hardness, and impact toughness; (c) geometric defects (wrinkles, folds, burrs, underfill, ruptures) and fracture; (d) resistance to corrosion; (e) surface roughness; (f) formability and machinability, etc. Some of these changes take place in the rest of the production procedure and can be seen on the final products.

Mathematical models that describe the rheological behaviour of working material can be presented in a simplified manner using flow curves. Flow curves represent the elementary and most important source of information on the possible material behaviour in forming processes. Flow curves establish the relationship between flow stress and the so-called thermo-mechanical forming process factors.

Currently, flow curves utilized to represent the rheological behaviour of working material in hot conditions do not take into account mechanical properties and microstructure before the processing, as well as strain path and variability of strain rate during the forming process. Real forming processes are, as a rule, sequential, unproportional, and unmonotonic. The effect of these phenomena is currently being studied intensively.

Mathematical models that describe the rheological behaviour of working material belong to the following three types [1]: (a) empirical-analytical models; (b) physically-based models; (c) empirical non-analytical models.

The empirical-analytical models are used more than the other types, because they make it easy to identify the constitutive material coefficients and it is also easy to implement them in FEM analysis. The other mathematical models require specific scientific and computational knowledge, large experiment, complex analysis, and time-consuming procedures for the calculation constitutive material coefficients. The main disadvantage of the first type of mathematical models is that they require a priori an assumption on the kind of constitutive law.

For the purposes of engineering calculation, it is convenient to use graphs, while it is necessary to display flow curves in the analytical form for the purposes of numerical simulations of forming processes [2], [3].

Mathematical models can be expressed in different formulae. Generally, the predictive capability and usefulness of a mathematical model can be assessed on the basis of the accuracy, consistency, transferability, and versatility of its structure [1], [4]. On the other hand, the availability of reliable experimental data is often a decisive factor whether one or another model should be used.

This paper is not intended to provide a detailed and systematic review of flow curve models that are known and can be found in the literature. In comprehensive papers [1], [8], various methodologies for the prediction of the flow stress in hot forming processes are presented. In the last several years, this topic has been intensively studied. Numerous papers relate to the modifications of well-known flow curves for different types of materials [5], [6], [7], [8].

The principal objective of the present paper is rather to choose a simple and sufficiently accurate mathematical description of the flow curve, which can be easily used in engineering calculations and numerical simulations. If numerical errors are neglected, the accuracy and correctness of numerical analysis and simulation depend, to a great extent, on the flow curve model.

For the abovementioned reasons, the response of materials to changeable hot forming conditions is very complex and specific. A generalized equation that describes the material behaviour during a hot forming process can be written in the implicit form:

$$F(\sigma_e, \varphi, \dot{\varphi}, T, t) = 0 \tag{1}$$

where σ_e – is the flow stress (equivalent normal stress) , φ – is the strain, $\dot{\varphi}$, – are the strain rate, T – is the workpiece temperature and t – is the time.

The determination of flow curves is usually based on uniaxial upsetting, uniaxial tension or torsion test. Each of these simple tests has severe shortcomings and cannot simulate the real forming processes in a proper way. Therefore, a general flow curve model may be expressed in the following explicit form:

$$\sigma_e = f(\varphi, \dot{\varphi}, T) \tag{2}$$

Some important flow curve models of empirical-analytical type are given below [1], [9], [10], [11], [12]:

$$\sigma_e = C \cdot \varphi^n \cdot \dot{\varphi}^m \cdot T^q \tag{3a}$$

$$\sigma_e = C \cdot e^{(n \cdot \varphi + m \cdot \dot{\varphi} + q \cdot T)} \tag{3b}$$

$$\sigma_e = C \cdot \varphi^n \cdot \dot{\varphi}^m \cdot e^{q \cdot T} \tag{3c}$$

$$\sigma_e = C \cdot \varphi^n \cdot \dot{\varphi}^m \cdot e^{q / T} \tag{3d}$$

$$\sigma_e = C \cdot \varphi^{(n+q \cdot T)} \cdot e^{(n_1 \cdot \varphi)} \cdot \dot{\varphi}^{(m+q_1 \cdot T)} \cdot e^{q_2 \cdot T} \tag{3e}$$

$$\sigma_e = \left(A + B \cdot \varphi^n\right) \cdot \left(1 + C \cdot \ln \frac{\dot{\varphi}}{\dot{\varphi}_0}\right) \cdot \left[1 - \left(\frac{T - T_0}{T_m - T_0}\right)^q\right] \tag{3f}$$

$$\sigma_e = \frac{1}{\alpha} \sinh^{-1}\left[\frac{1}{D} \cdot \dot{\varphi} \cdot \exp\left(\frac{Q}{R \cdot T}\right)\right]^{1/n} \tag{3g}$$

where $\dot{\varphi}_0$ – is the reference strain rate, T_0, T_m – are the reference temperature and melting temperature of workpiece material, respectively, C – is the strain/strain rate sensitivity constant, n, m, q – are strain hardening exponent, strain rate hardening exponent, and thermal softening exponent, respectively, A – is the yield stress at a reference strain and temperature, B – is the strain hardening modulus, D, α – are constants, Q – is the activation energy, R – is the universal gas constant.

In the present study, four types of flow curves (3a-3d) are explored in detail. These equations do not include all physical phenomena present in forming processes, such as strain hardening and softening, strain history, as well as recovery and recrystallization with microstructural evolution. A complete coverage of all of these phenomena with a single flow curve model is an extremely difficult theoretical and practical problem. On the other hand, these flow curve models are simple and can be easily analyzed by the regression method.

This paper consists of two main parts. The first one deals with basic postulates and advantages of the design of experiment. In the second part, the selected mathematical models for describing the flow curves in hot forming processes are presented, explored, discussed, and compared with each other.

2. DESIGN OF EXPERIMENT

2.1 Overview

The design of experiment (DoE) addresses the preparation, physical realization of the experiment, processing and analysis of the experimental data according to the previously determined plan, which enables the variations of factors simultaneously on various levels, in each following trial. In that sense, the DoE represents a qualitatively new approach to the theoretical-experimental analysis and optimization of complex processes/ systems, with universal application and range of advantages in comparison to the concept and practice of the one-factor-at-a-time method [13], [14], [15], [16].

In the DoE there are various methods developed for solving different and complex research tasks as: (a) mathematical modelling of phenomena, complex processes and systems in space and time, (b) study of internal mechanisms of various phenomena, (c) optimization and optimal control of process/systems.

Each of the abovementioned tasks can be solved on its own, regardless of others. However, in many cases all of them are treated as an integral research task.

The basic characteristics and advantages of the DoE (especially becoming evident in complex research objects, with a large number of factors) are as follows [13], [14], [16]: (a) minimum number of needed trials, (b) maximum amount of information from the given number of trials, (c) successively conducting experiment in stages (step by step), from simpler to more complex designs, (d) simple statistical (regression and dispersion) analysis of experimental data, (e) possibility of qualitative and quantitative assessment of effects of each factor (perhaps, their interaction) on the target function, due to variation of factors simultaneously, (f) easy optimization of a process/system which is the subject of research, on the basis of obtained empirical (regression) model of the target function, that encompasses the entire experimental space, (g) minimum time and material losses (expenses) for experiment realization, (h) eliminating the subjective influence of the researcher, etc.

The outline of the recommended procedure in the DoE, as a statistical approach in designing and analyzing an experiment, include [13], [17]: (a) recognition and statement of the problem, (b) choice of factors, levels, and range, (c) selection of the response variable, (d) choice of experimental design, (e) performing the experiment, (f) statistical analysis of the data, (g) conclusions and recommendations.

The mathematical model represents a formal and analytical expression of physical, geometrical and other characteristics of a real process/system. The selection of the mathematical model, which is used to establish a connection between factors (inputs) and target function (output), depends on the goal of the research, complexity of the phenomenon being researched, and the selected experimental design. Quantity and quality of information on the subject of research also influence the selection of mathematical model, as well as the knowledge of experimental techniques by the experimenter. Therefore, the selection of mathematical model, which is conducted in the initial phase of experimental design, represents its most significant segment, regardless of the ultimate goal of the undertaken research.

The quality and accuracy of a mathematical model, that is, regression equation, is defined by the complexity of the selected function. Theory and practice have shown that in most cases the best choice is a mathematical model in the form of polynomials (linear, quasi-linear, square, etc.). More complex mathematical models ensure higher accuracy in the prediction of researched process/system behaviour in the selected hyper-space. However, such models also imply a more complex experimental procedure, and more complex and time-consuming analysis and interpretation of experimental results, as well as greater time losses, material expenses, etc.

On the other hand, the selection of the simplest-linear-mathematical model does not enable the analysis of the effects of factor interactions. However, various examples undoubtedly show that the effects of factor interactions on the target function can be more significant than the effects of single main factors.

It should also be emphasized that it is the case of lower-order factor interactions, since the higher-order factor interactions can be neglected. Namely, interactions of many factors, as a rule, do not influence the accuracy of the selected mathematical model.

Various experiments and practical knowledge point to the fact that the mathematical models usable for the analysis and optimization of complex processes/systems are those in the form of multiple power, exponential, or some other function instead of polynomials. Such mathematical models are easily transformed into linear functions (first order polynomials). As it is known, in the DoE the procedure of statistical data processing (regression and dispersion analysis) is entirely and unambiguously defined for such mathematical models.

The choice of appropriate criterion for building the mathematical model is not often obvious [18], [19]. It is understood that there is enough reliable information about the research object so that an adequate form of the mathematical model can be chosen with sufficient certainty.

2. 2. Mathematical model in the form of multiple power function

Numerous experiments and practical knowledge have shown that a successful modeling of the most diverse technological processes/systems can be made using a multiple power equation in the following form [20], [21], [22], [23]:

$$F_c = C \cdot X_1^{p_1} \cdot X_2^{p_2} \cdot X_3^{p_3} \cdots X_i^{p_i} \cdots X_k^{p_k} = C \cdot \prod_{i=1}^{k} X_i^{p_i} \qquad (4)$$

where X_i is the natural factors (natural coordinates), C and p_i are constants to be determined, k is the number of factors.

By logarithming, the target function (4) is formally reduced to the linear form (5a) or (5b):

$$\ln F_c = \ln C + p_1 \cdot \ln X_1 + p_2 \cdot \ln X_2 + p_3 \cdot \ln X_3 + \cdots + p_k \cdot \ln X_k = \ln C + \sum_{i=1}^{k} p_i \cdot \ln X_i \qquad (5a)$$

$$y = b_0 + b_1 \cdot x_1 + b_2 \cdot x_2 + b_3 \cdot x_3 + \cdots + b_k x_k = b_0 + \sum_{i=1}^{k} b_i \cdot x_i \qquad (5b)$$

where $y = \ln F_c$; $b_0 = \ln C$; $b_i = p_i$; $x_i = \ln X_i$.

Multiple linear regression equation (5b) is represented in coded coordinates. The connection between natural and coded coordinates is established through the following transformation equations [17], [21], [23]:

$$x_i = 2\frac{\ln X_i - \ln X_{i\max}}{\ln X_{i\max} - \ln X_{i\min}} + 1; \; i = 1, 2, 3....k \qquad (6)$$

For the purposes of dispersion analysis it is necessary to repeat trials at certain points in the experimental hyper-space. The systems of trial repetition are as follows:

1. Repetition for n_0 times only in the central point of the experimental design ($x_i = 0$);
2. Uniform repetition for n times in each vertex of an experimental hyper-cube ($x_i = \pm 1$);
3. Non-uniform repetition for n_u times in certain points of an experimental hyper-cube, or only in one point.

The repetition according to the third option is applied when the trials for a specific combination of process factors are very expensive and/or time consuming (if not impossible in manufacturing facilities).

It should be emphasized that for such a mathematical model the determination of basic levels of factors is conducted by applying the following relations:

$$X_{i0}^2 = X_{i\,max} \cdot X_{i\,min} \tag{7}$$

One should keep in mind that the mathematical processing of the experimental data is performed in the same way as the application of the mathematical model in the polynomial form.

After determining the coefficients of linear regression (5) in a widely known manner, the unknown constants of the target function (4) are calculated using the following formulae [17]:

$$p_i = \frac{2b_i}{\ln(X_{i\,max} / X_{i\,min})} \tag{8}$$

$$C = \exp\left(\sum_{i=0}^{k} b_i - \sum_{i=1}^{k} p_i \ln X_{i\,max}\right) \tag{9}$$

2.3. Mathematical model in the form of multiple exponential function

For an initial form of the mathematical model (target function) the multi exponential function can be chosen [17]:

$$F_c = C \cdot e^{(p_1 \cdot X_1 + p_2 \cdot X_2 + p_3 \cdot X_3 + \cdots + p_k \cdot X_k)} = C \cdot \exp\left(\sum_{i=1}^{k} p_i \cdot X_i\right) \tag{10}$$

By analogous procedure for this function the following equations are obtained:

$$\ln F_c = \ln C + p_1 \cdot X_1 + p_2 \cdot X_2 + p_3 \cdot X_3 + \cdots + p_k \cdot X_k = \ln C + \sum_{i=1}^{k} p_i \cdot X_i \tag{11a}$$

$$y = b_0 + b_1 \cdot x_1 + b_2 \cdot x_2 + b_3 \cdot x_3 + \cdots + b_k x_k = b_0 + \sum_{i=1}^{k} b_i \cdot x_i \tag{11b}$$

$$x_i = 2\frac{X_i - X_{i0}}{X_{i\max} - X_{i\min}};\ i = 1, 2, 3....k \tag{12}$$

$$X_{i0} = \frac{X_{i\max} + X_{i\min}}{2} \tag{13}$$

$$p_i = \frac{2b_i}{X_{i\max} - X_{i\min}} \tag{14}$$

$$C = \exp\left(b_0 - \sum_{i=1}^{k} p_i \cdot X_{i0}\right) \tag{15}$$

2. 4. Mathematical model in the form of multiple power-exponential function

2.4.1. Variant I

In order to analyze certain complex processes and systems, it is necessary to choose the function in the following form, as an initial mathematical model [17]:

$$F_c = C \cdot X_1^{p_1} \cdot X_2^{p_2} \cdot X_3^{p_3} \cdots X_j^{p_j} \cdot e^{(p'_{j+1} \cdot X_{j+1} + p'_{j+2} \cdot X_{j+2} + p'_{j+3} \cdot X_{j+3.} + \cdots + p'_k \cdot X_k)} = C \cdot \prod_{i=1}^{j} X_i^{p_i} \cdot \exp\left(\sum_{i=j+1}^{k} p'_i \cdot X_i\right) \tag{16}$$

For this function the following equations are valid:

$$\ln F_c = \ln C + \sum_{i=1}^{j} p_i \cdot \ln X_i + \sum_{i=j+1}^{k} p'_i \cdot X_i \tag{17a}$$

$$y = b_0 + \sum_{i=1}^{k} b_i \cdot x_i \tag{17b}$$

$$x_i = 2\frac{\ln X_i - \ln X_{i\max}}{\ln X_{i\max} - \ln X_{i\min}} + 1;\ \ x_i = 2\frac{X_i - X'_{i0}}{X_{i\max} - X_{i\min}} \tag{18}$$

$$X_{i0} = \left(X_{i\max} \cdot X_{i\min}\right)^{1/2}\ ; \qquad X'_{i0} = \frac{X_{i\max} + X_{i\min}}{2} \tag{19}$$

$$p_i = \frac{2b_i}{\ln(X_{i\max} / X_{i\min})}\ ;\ p'_i = \frac{2b_i}{X_{i\max} - X_{i\min}} \tag{20}$$

$$C = \exp\left(\sum_{i-0}^{k} b_i - \sum_{i=1}^{j} p_i \cdot \ln X_{i\max} - \sum_{i=j+1}^{k} p'_i \cdot X'_{i0}\right) \tag{21}$$

2.4.2. Variant II

In some cases, the following mathematical model can be adequate:

$$F_c = C \cdot X_1^{p_1} \cdot X_2^{p_2} \cdot X_3^{p_3} \cdots X_j^{p_j} \cdot e^{(p'_{j+1}/X_{j+1}+p'_{j+2}/X_{j+2}+p'_{j+3}/X_{j+3}+\cdots+p'_k/X_k} = C \cdot \prod_{i=1}^{j} X_i^{p_i} \cdot \exp\left(\sum_{i=j+1}^{k} p'_i / X_i\right) \quad (22)$$

For this function the following equations are valid:

$$\ln F_c = \ln C + \sum_{i=1}^{j} p_i \cdot \ln X_i + \sum_{i=j+1}^{k} p'_i / X_i \quad (23a)$$

$$y = b_0 + \sum_{i=1}^{k} b_i \cdot x_i \quad (23b)$$

$$x_i = 2\frac{\ln X_i - \ln X_{i\max}}{\ln X_{i\max} - \ln X_{i\min}} + 1; \quad x_i = 2\frac{X_{i\min}(X_i - X_{i\max})}{X_i(X_{i\max} - X_{i\min})} + 1 \quad (24)$$

$$X_{i0} = \left(X_{i\max} \cdot X_{i\min}\right)^{1/2}; \quad X'_{i0} = 2\frac{X_{i\max} \cdot X_{i\min}}{X_{i\max} - X_{i\min}} \quad (25)$$

$$p_i = \frac{2b_i}{\ln(X_{i\max}/X_{i\min})}; \quad p'_i = \frac{2b_i}{\left(X_{i\max} - X_{i\min}\right)/X_{i\max} \cdot X_{i\min}} \quad (26)$$

$$C = \exp\left(\sum_{i=0}^{k} b_i - \sum_{i=1}^{j} p_i \cdot \ln X_{i\max} + \sum_{i=j+1}^{k} p'_i / X_i\right) \quad (27)$$

3. EXPERIMENT

The experimentation was carried out on the torsional plastometer, in a single deformation step, at selected constant temperatures and constant strain rates. Alloyed steel 42CrMo4 (DIN/EN designation) was selected for exploration.

Three factors (input values, independent variables) were chosen for modelling the flow curves: shear strain (γ), shear strain rate ($\dot{\gamma}$) and temperature (T). Flow stress (σ_e) was chosen for the target function (output value, dependent variable).

For the purposes of this research, a data subset of eight trials was extracted from a large experiment (Table 1) [24], and was used for conducting the regression analysis. Other trials were used for the verification of the selected mathematical models. The selected data subset (1, 6, 31, 36, 73, 78, 103, 108) formed the vertices of the experimental cube and, in this way, the entire experimental space was covered. Therefore, a full two-level factorial design 2^3 was formed [13], [14], with eight factor combinations (Table 2).

Table 1. Experimental data set for flow stress in various forming conditions

σ_e (N/mm^2)							
42CrMo4 (Č.4732)		**T** (°C)					
$\dot{\gamma}$ (s^{-1})	γ (–)	**800**	**900**	**1000**	**1100**	**1200**	**1250**
1.26	**0.374**	(1) 202.44	(2) 157.95	(3) 86.47	(4) 54.76	(5) 38.82	(6) 34.18
	0.752	(7) 204.86	(8) 163.02	(9) 88.69	(10) 57.28	(11) 41.41	(12) 35.26
	1.128	(13) 198.43	(14) 153.30	(15) 83.11	(16) 53.36	(17) 38.03	(18) 32.91
	1.504	(19) 187.34	(20) 143.08	(21) 79.07	(22) 51.39	(23) 36.85	(24) 31.94
	1.880	(25) 175.60	(26) 133.92	(27) 76.16	(28) 50.33	(29) 35.98	(30) 31.28
	2.256	(31) 165.56	(32) 126.92	(33) 73.71	(34) 49.35	(35) 35.10	(36) 30.91
5.03	**0.374**	(37) 243.17	(38) 164.60	(39) 102.98	(40) 73.60	(41) 48.25	(42) 42.61
	0.752	(43) 253.71	(44) 175.35	(45) 110.68	(46) 81.26	(47) 54.31	(48) 46.11
	1.128	(49) 249.20	(50) 170.84	(51) 105.53	(52) 78.84	(53) 51.53	(54) 44.41
	1.504	(55) 240.93	(56) 163.75	(57) 100.16	(58) 74.62	(59) 48.41	(60) 42.73
	1.880	(61) 232.63	(62) 156.71	(63) 95.13	(64) 71.03	(65) 45.30	(66) 41.30
	2.256	(67) 226.31	(68) 144.49	(69) 89.85	(70) 67.94	(71) 43.42	(72) 39.82
7.55	**0.374**	(73) 263.58	(74) 181.18	(75) 111.48	(76) 81.62	(77) 51.17	(78) 46.43
	0.752	(79) 273.65	(80) 199.86	(81) 120.06	(82) 87.96	(83) 57.50	(84) 49.86
	1.128	(85) 274.24	(86) 200.07	(87) 119.91	(88) 90.38	(89) 56.83	(90) 52.35
	1.504	(91) 268.15	(92) 189.20	(93) 112.30	(94) 83.13	(95) 53.25	(96) 49.30
	1.880	(97) 262.36	(98) 179.41	(99) 106.00	(100) 78.43	(101) 51.43	(102) 47.45
	2.256	(103) 253.60	(104) 167.26	(105) 101.75	(106) 75.34	(107) 49.46	(108) 45.69

Table 2. Plan-matrix for non-linear mathematical model (multiple power function)

Natural factors	$X_1 = \gamma$	$X_2 = \dot{\gamma}$	$X_3 = T$	Response $y = K = C \cdot \gamma^{n} \cdot \dot{\gamma}^{m} \cdot T^{q}$		
High level	2.256	7.55	1250			
Middle level	0.921	3.08	1000			
Low level	0.376	1.26	800			
Coded factors / Trials	x_1	x_2	x_3	Measurement		Calculation
				y	$\ell n\, y$	$\hat{y}$
1	+1	+1	+1	45.69	3.82188	44.0137
2	- 1	+1	+1	46.43	3.83795	48.1153
3	+1	- 1	+1	30.91	3.43108	31.1153
4	- 1	- 1	+1	34.18	3.53164	34.0115
5	+1	+1	- 1	253.60	5.53576	247.4974
6	- 1	+1	- 1	263.58	5.57436	270.5346
7	+1	- 1	- 1	165.56	5.10933	174.9500
8	- 1	- 1	- 1	202.44	5.31044	191.2535

4. DETERMINING THE CONSTITUTIVE MATERIAL COEFFICIENTS

4.1. Mathematical model in the form of multiple power function

The determination of the parameters of a mathematical model, i.e. the unknown coefficients of regression equation, in DoE for a full/fractionated two-level factorial design is well-known and fully established [13], [14]. In this case, on the basis of the experimental data set in Tab. 2, the following is obtained:

$$b_0 = 4.02143;\ b_1 = -0.04454;\ b_2 = 0.04294;\ b_3 = -0.46447 \qquad (28)$$

On the basis of formulae (8) and (9) the following constants are calculated:

$$p_1 = -0.04972;\ p_2 = 0.19373;\ p_3 = -3.86935;\ C = 2979.3 \cdot 10^{10} \qquad (29)$$

By using the transformation equation (6), the non-linear multiple regression equation, as a power function, obtains the following form:

$$\sigma_e = 2979.3 \cdot 10^{10} \cdot \gamma^{-0.04972} \cdot \dot{\gamma}^{0.19373} \cdot T^{-3.86935} \qquad (30a)$$

Notice: For the purposes of using equations from Chapter 2.2, the following substitution of variables was introduced: $F_c \rightarrow \sigma_e;\ X_1 \rightarrow \gamma;\ X_2 \rightarrow \dot{\gamma};\ X_3 \rightarrow T;\ p_1 \rightarrow n;\ p_2 \rightarrow m;\ p_3 \rightarrow q$.

Since relations $\gamma = \sqrt{3}\,\varphi$ and $\dot{\gamma} = \sqrt{3}\,\dot{\varphi}$ are valid, Eq. (30a) may be transformed in the form:

$$\sigma_e = 3224.6 \cdot 10^{10} \cdot \varphi^{-0.04972} \cdot \dot{\varphi}^{0.19373} \cdot T^{-3.86935} \qquad (30b)$$

The first-order mathematical model can be expanded by introducing interactions of the main design factors. However, the non-linear mathematical models in the form of complex power functions with interactions do not guarantee higher accuracy than the same models without interactions [17], [23]. On the other hand, such mathematical models make intricate the analysis and interpretation of modeling results. The application of such mathematical models can be justified only in specific cases.

4.2. Other mathematical models

Based on the previously described procedure, the following regression equations can be derived:
a) multiple exponential function,
b)

$$\sigma_e = 3922.6 \cdot e^{(-0.04738 \cdot \gamma + 0.05514 \cdot \dot{\gamma} - 0.00384 \cdot T)} \tag{31a}$$

$$\sigma_e = 3922.6 \cdot e^{(-0.08206 \cdot \varphi + 0.09551 \cdot \dot{\varphi} - 0.00384 \cdot T)} \tag{31b}$$

c) multiple power-exponential function (Variant I),
d)

$$\sigma_e = 3762.2 \cdot \gamma^{-0.04972} \cdot \dot{\gamma}^{0.19373} \cdot e^{-0.00384 \cdot T} \tag{32a}$$

$$\sigma_e = 4071.9 \cdot \varphi^{-0.04972} \cdot \dot{\varphi}^{0.19373} \cdot e^{-0.00384 \cdot T} \tag{32b}$$

e) multiple power-exponential function (Variant II),
f)

$$\sigma_e = 0.5045 \cdot \gamma^{-0.04972} \cdot \dot{\gamma}^{0.19373} \cdot e^{6271/(273+T)} \tag{33a}$$

$$\sigma_e = 0.5460 \cdot \varphi^{-0.04972} \cdot \dot{\varphi}^{0.19373} \cdot e^{6271/(273+T)} \tag{33b}$$

5. RESULTS AND DISSCUSION

All of the abovementioned models indicate that flow stress decreases with increasing temperature and strain, and decreasing strain rate. On the basis of the regression coefficient values it can be concluded that the forming temperature has the most pronounced impact on the flow stress, while the degree of influence of strain and strain rate is almost identical. That is an expected response of steel in hot forming processes [25], [26], [27]. In this case, it is easy to prove that factor interactions have negligible influence on the flow stress.
The current study concerns a single replicated experimental design (sometimes called unreplicated factorial), because it has only one trial at each factor combination. Replication reflects both sources of variability between trials and (potentially) within trials and provides an estimate of ’’pure error’’. Therefore, with only one replicate, there is no possibility of carrying out dispersion analysis in a full mathematical model [13], [14], [28]. For that reason, the evaluation of the adequacy of the mathematical model cannot be performed directly.

When a single replicated experiment is conducted, in the presence of great response variability, there is a risk of fitting the mathematical model to noise, which could consequently result in misleading conclusions. That was not the case in this study.
Here, the verification of the given mathematical models (regression equations) can be done in an indirect way using the remaining experimental results in Tab. 1.
For estimating the adequacy of all regression equations, three statistical parameters were employed, correlation coefficient (R), mean absolute percentage error ($\overline{\Delta}$), and maximal absolute percentage error (Δ_{max}). The following relations were used for the calculation of errors:

$$\Delta = \left| \frac{y_{cal} - y_{exp}}{y_{exp}} \right| 100(\%) \; ; \; \overline{\Delta} = \frac{\sum \Delta}{n} \qquad (34)$$

where n is the number of trials in the experiment.
The results of calculation are presented in Table3.

Table 3. Comparison of the flow curve models

Statistical parameters	Eq. (30)	Eq. (31)	Eq. (32)	Eq. (33)
Δ_{max} (%)	18.83	13.88	18.19	24.19
$\overline{\Delta}$(%)	5.82	5.63	5.71	7.44
R	0.994	0.995	0.995	0.991

On the basis of the data from Table 3, one can conclude that the prediction errors are relative small for all generated multiple regression equations. Based on Fig. 1 (which relates to the entire experiment), it can be concluded that all mathematical models ensure high levels of correlation, which are very similar, although not identical. The perfect prediction implies that all points should lie on a straight line passing through the origin and inclined at 45^0. From Fig. 1 one can observe that there is a relatively small deviation of the line of regression (which represents the best linear approximation of the data) from the ideal line. Taking three criteria ($\Delta_{max}, \overline{\Delta}, R$) into consideration, it is obvious that Eq. (31) represents the best-fit flow curve model.
Differences between the measured and predicted flow stresses can be a consequence of theoretical assumptions, experimental errors, variations of forming process factors, strain history, geometric deviations of samples, failures in material and variability of its mechanical properties, metallurgical background, and other phenomena that occur in material during deformation. It is not easy to quantify separately these sources of inaccuracy.
This study confirms that by selecting the adequate type of experimental design and mathematical model it is possible to reduce significantly the number of necessary trials.

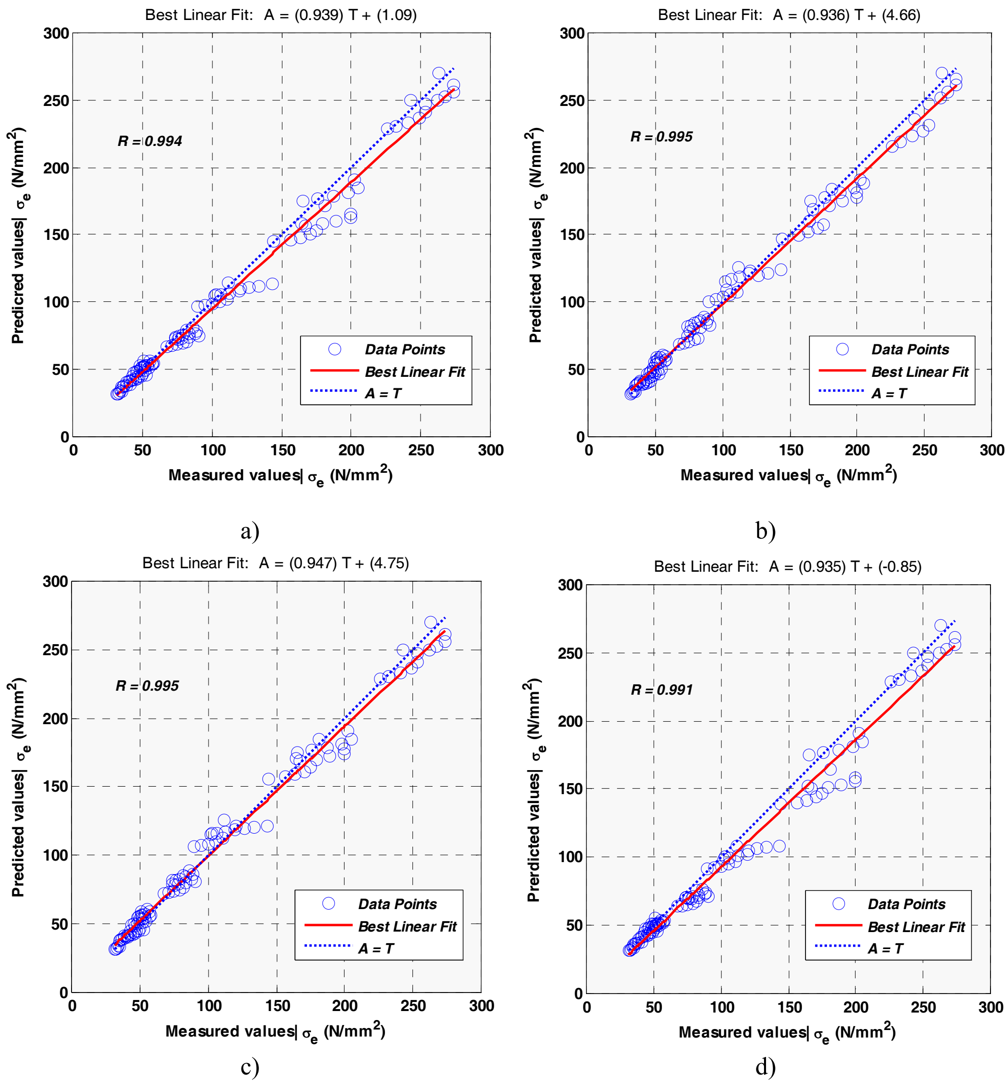

Fig. 1. - Correlation between measured and predicted flow stress for whole data set:
a) Eq. (30) ; b) Eq. (31) ; c) Eq (32) ; d) Eq. (33)

6. CONCLUSION

In this paper, some of flow curve models were explored and discussed. It was found that a flow curve for alloyed steel, in hot forming condition, could be adequately approximated using a multiple exponential function.

The results of the performed analysis show that all basic forming process factors (temperature, strain, strain rate) are significant. Forming temperature is the dominant factor affecting the flow stress, followed by strain and strain rate.
The predicted flow stress values of 42CrMo4 steel are in good agreement with experimental results, especially when both forming temperature and strain are high. These forming conditions are typical for most of real forming processes. Proposed constitutive equation can be used to design hot forming processes and numerical simulation for this high-strength alloyed steel.

ACKNOWLEDGEMENT

This work was carried out within the project TR 35034 financially supported by the Ministry of Education and Science of the Republic of Serbia.

REFERENCES

[1] Bariani P.F., Dal Negro T., Bruschi S.: Testing and modelling of materials response to deformation in bulk metal forming, CIRP Annals, Vol. 53/2, 2004, pp. 573-596.

[2] Movrin D., Plančak M., Vilotić D., Milutinović M., Skakun P., Lužanin O., Trbojević I.: Optimization and design of multistage hot forging processes by numerical simulation and experimental verification, Journal for Technology of Plasticity, Vol. 35, 2010, pp. 75-89.

[3] Mandić V., Živković M.,Vulović S., Marinković T.: FEM analysis for the extrusion process of tubes using porthole dies, Journal for Technology of Plasticity, Vol. 29, 2004, pp. 35-43.

[4] Murthy A., Lenard J.G.: Statistical evaluation of some hot rolling theories. Transaction of the ASME, Journal of Engineering Materials and Technology, Vol. 104, pp. 47-52.

[5] Lin Y.C, Liu G.: A new mathematical model for predicting flow stress of typical high-strength alloy steel at elevated high temperature, Computational Materials Science, Vol. 48, 2010, pp. 54-58.

[6] Sung J.H., Kim J.H., Wagoner R.H.: A plastic constitutive equation incorporating strain, strain-rate, and temperature, International Journal of Plasticity, Vol. 26, 2010, pp. 1746-1771.

[7] Chunlei G., Yongdong X., Mengjun W.: Prediction of the flow stress of Al6061 at hot deforming conditions, Materials Science and Engineering: A, Vol. 528, 2011, pp. 4199-4203.

[8] Lin Y.C., Chen X-M.: A critical review of experimental results and constitutive descriptions for metals and alloys in hot working, Materials and Design, Vol. 32 , 2011, pp. 1733-1759.

[9] Grosman F.: Application of a flow stress function in programs for computer simulation of plastic working processes, Journal of Materials Processing Technology, Vol. 64, 1997, pp. 169-180.

[10] Gronostajski Z.: The constitutive equations for FEM analysis, Journal of Materials Processing Technology, Vol. 106, 2000, pp. 40-44.

[11] Johnson G.R., Cook W.H.: Fracture characteristic of three metals subjected to various strains, strain-rate, temperature and pressure, Engineering Fracture Mechanics, Vol. 21, 1985, pp. 31–48.

[12] Heller C., Schmoeckel D.: Umformen von Aluminium-blechen bei erhöhten Temperaturen, Aluminium, Vol. 64, No. 4, 1988, pp. 398-405. (In German).

[13] Montgomery D.C.: Design and analysis of experiments, John Wiley & Sons, Inc., New York, 2001.

[14] Novik F.S., Arsov, J.B.: Optimization of technological processes using methods of experimental design, Mashinostoenie, Moscow, 1980. (In Russian)

[15] Czitrom V.: One-factor-at-a-time versus designed experiments, The American Statistician, Vol. 53/2, 1999, pp. 126-131.

[16] Marinković V.: Rationalization of experimental investigation using theory of experimental design."IMK-14", Vol. XIII, No. 1/2, 2007, pp. 23-36. (In Serbian).

[17] Marinković V.: Application of some non-linear mathematical models to the theory of experimental design, Facta Universitatis, Series: Mechanical Engineering, Vol.1, No. 1, 1994, pp. 103-117.

[18] Montgomery, D.C., Myers, R.H., Carter, W.H., and Vining G.G.: The hierarchy principle in designed industrial experiments, Quality and reliability engineering international, Vol. 21, 2005, pp. 197-201.

[19] Peixoto, J.L.: A property of well-formulated polynomial regression models, The American Statistician, Vol. 44/1, 1990, pp. 26-30.

[20] O'Brien P., Thompson W., Russell J.K.: A statistical model for predicting surface texture of machined thermo plastics, CIRP Annals, Vol. 25, 1977, pp. 339-344.

[21] Choudhury I.A., El-Baradie M.A. Tool-life prediction model by design of experiment for turning high strength steel (290 BHN), Journal of Material Processing Technology, Vol. 77, 1998, pp. 319-326.

[22] Dasgupta R., Thakur R., Govindrjan B.: Regression analysis of factors affecting high stress abrasive wear behaviour, Practical Failure Analysis, Vol. 2/2, 2000, pp. 65-68.

[23] Lazarević A., Marinković V., Lazarević D.: Expanded non-linear mathematical models in the theory of experimental design: A case study, 10th Inter. Conf.-RaDMI 2010, Donji Milanovac, 2010, pp. 304-310.

[24] Kovačević R.: Contribution to the identification of the formability of steels with the choice of optimal methods for the examination of their plastic properties, Faculty of Mechanical Engineering, Titograd, 1978. (In Serbian).

[25] Ximbo L., Hongsheng X., Zhiliang Z.: Flow stress of carbon steel 08F in temperature range of warm-forging, Journal of Material Processing Technology, Vol. 139, 2003, pp. 543-546.

[26] Mirzadeh H., Najafizadeh A.: Extrapolation of flow curves at hot working conditions, Materials Science and Engineering: A, Vol. 527, 2010, pp. 1856-1860.

[27] Marinković V.: Analysis of the influental factors on the flow stress in the warm forming processes by applying latin square design. 9th International Conference-DEMI 2009, Banja Luka, 2009, pp.163-168.

[28] Costa N., Pereira Z.L.: Objective methods for analyzing unreplicated factorial designs, Proceedings of the World Congress on Engineering-WCE 2007, Vol. II, London, 2007.

PREDVIĐANJE NAPONA TEČENJA LEGIRANOG ČELIKA PRI TOPLOM DEFORMISANJU PRIMENOM RAZLIČITIH MATEMATIČKIH MODELA I PLANIRANJA EKSPERIMENTA

Velibor Marinković

Univerzitet u Nišu, Mašinski fakultet, Aleksandra Medvedeva 14, Niš, Srbija

REZIME

U poslednjih nekoliko decenija razvijen je veliki broj matematičkih modela za opisivanje reološkog ponašanja metala i legura u uslovima toplog deformisanja. Korišćeni su mnogi različiti pristupi za određivanje konstitutivne jednačine, tj. jedne složene nelinearne relacije između napona tečenja i različitih faktora procesa deformisanja. U ovom radu su istraživani neki relativno jednostavni matematički modeli za predikciju napona tečenja legiranog čelika tokom procesa toplog deformisanja. Da bi se modelovao konstitutivni odziv materijala, napon tečenja je određen kao funkcija glavnih faktora, kao što su stepen deformacije, brzina deformacije i temperatura. Nakon analize rezultata proračuna izabran je konstitutivni zakon, koji reprezentuje najbolji model aproksimacije krive tečenja. Apsolutne procentualne greške i koeficijent korelacije ukazuju da predložena kriva tečenja može predviđati napon tečenja u realnim uslovima deformisanja sa dobrom korelacijom i generalizacijom. Takođe, ova kriva tečenja se može lako koristiti u inženjerskim proračunima i numeričkim simulacijama.

Ključne reči: *krive tečenja, toplo deformisanje, planiranje eksperimenta, legirani čelik*