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IDENTIFICATION OF STRESS STATE IN FORMED TUBULAR BILLETS

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ABSTRACT

In this paper stress state was determined for tubular billets, using differential equations for equilibrium of volume element and conditions of plastic flow within the restrictions of ratio between normal and tangential stresses on outer and inner surface.

Experimental investigations of forming tubular billets are entirely in conformance with FEM numerical simulation which was done using CAMPform 2D software.

Key words: Stress state, Tubular billets, Differential equations, FEM simulation

1. INTRODUCTION

Analysis of stress state is shown on an example of forming of a tubular billet. The aim of this experimental investigation is to verify theoretical assumptions as well as to identify basic process parameters. Maintaining stability of the forming process often requires analysis of stress-strain state along the deformation zone. The magnitude of flow stress can be determined either experimentally or by solving a system of volumetric stress state equations for characteristic locations of the deformation zone.

One of essential theoretical methods for determination of stress state in a specific cross-section of a deformed tubular element - which requires knowledge of material properties (factor of hardening) - is the method of solving differential equations of equilibrium for the state of plastic flow. This method also requires boundary conditions expressed as the ratio between outer- and inner-wall stresses of the deformed billet.

The differential equations of stress state are often complex, which is why certain simplifications are introduced. These simplifications depend on the billet dimensions and shape of the tool which defines the forming process in the experiment.

2. STRESS STATE OF VOLUME ELEMENT IN CHARACTERISTIC CROSS-SECTIONS ALONG DEFORMATION ZONE

Using tubular billet shown in Fig. 1 coordinate grid was created by segmenting billet height and circumference. The billet underwent deformation causing the grid to distort as the result of the distortion of the volume element in various increments along the deformation zone.

The predominant stress state in the volume element undergoing necking is a combination of three upsetting stresses.

Maximum deformation within the deformation zone is the radial upsetting which leads to wall thickening. Measurements have shown that the wall thickening is largest at the fillet radius where the spherical section transforms into the conic section (zone 4). Going further towards the end of the conic section, wall thickness starts to decrease regardless of the continuous diameter reduction, i.e. progressive necking.

This implies that, under forming, the material acts like a viscous medium, undergoing certain amount of relaxation at the end of the deformation zone, which results in reduced wall thickness.



Figure 1 - Distortion of the deformation zone on the billet

Figure 2 shows a diagram of a two-part tool engaging the billet during forming. The normal component of the necking force \mathbf{F}_{su} and the contact friction $\boldsymbol{\mu}$, generate friction force \mathbf{F}_t via shaping rings. The friction force causes stress state in the cylindrical section of the billet which is sufficient to provide instability, i.e. forming of the spherical section (the necking phase) [1], [2]. This force causes the reduction of billet cross-section, which results in an axial-symmetrical upsetting stress state. The shape of the deformation zone influences the stress-strain state as well as the boundaries of the forming process.



Figure 2 - Forming of axial instability during deformation phases

The stress state in a combined forming process of necking and expanding can be reviewed through phases of the deformation zone, as shown in Fig. 2:

- 1 free upsetting zone of the cylindrical section of the tubular billets
- 2 -deformation zone outside the contact area
- 3 -necking zone in the spherical section of the necking ring
- 4 -bending zone on the radius
- 5 -necking zone in the conical section

Volume element on the cylindrical section of the tubular workpiece (z_i) shown in Fig. 3 is loaded with force F_i which is offset by λ from the axis. This load can be decomposed into a concentric force F_i and a bending torque $M_i = F_i \cdot \lambda$ acting in the longitudinal cross-section plain.

The magnitude and the character of the load $p(z_I)$ can explain the occurrence of the double bulge on the cylindrical section, visible on the defective workpiece in Fig. 2. One concludes that proper transformation of the cylindrical section into spherical shape depends on the magnitude of force F_I and length z_I for certain parameters and type of material of steel tubular billets.

From the condition of force equilibrium in a tubular element loaded with torque M_1 , we have:

$$q(z_1) = \frac{2M_1}{(z_1 - dz_1) \times dz_1} \tag{1}$$

or, expressed as $p(z_1)$ per unit area on length (z_1)

$$p(z_1) = \frac{q(z_1)}{2\eta \cdot \pi} \tag{2}$$

Substituting $M_I = F_I \cdot \lambda$ and $F_I = F_{su}$ there follows:

$$p(z_1) = \frac{F_{su} \cdot \lambda}{r_1 \pi \cdot (z_1 - dz_1) \cdot dz_1}$$
(3)

which acts on the inner side, causing instability of the cylindrical section of the workpiece shown in Fig. 2. a, b, c.

From the condition of equilibrium of radial forces in a volume element (Fig. 3.), there follows:

$$p(z_1) \cdot r_1 \cdot dz_1 \cdot d\varphi_1 - 2\sigma_{\varphi_1} \cdot s_1 \cdot dz_1 \cdot \sin\frac{d\varphi_1}{2} = 0$$
⁽⁴⁾

which leads to conclusion that the ratio of radial and circular stresses equals:

$$p(z_1) = \sigma_{\varphi 1} \cdot \frac{s_1}{r_1} \,. \tag{5}$$



Figure 3 - Load approximation for the cylindrical section of the workpiece and the stress state of the volume element whose length is dz_1

From the condition of equilibrium of axial forces there follows:

$$\sigma_{z1} = \frac{F_1}{(D_1 - s_1) \cdot s_1 \cdot \pi} \tag{6}$$

Based on the experimental example of forming tubular workpieces, we can establish the magnitude of the necking force from the diagram of force change which signals the beginning of formation of the spherical section in the middle part of the cylindrical section of the deformation zone. Thus we derive σ_{zI} = -374.2 Mpa.

If σ_{z1} and $\sigma_{\varphi 1}$ are the main normal stresses, according to the hypothesis of maximum shear stress, plastic forming occurs when $\sigma_{z1} - \sigma_{\varphi 1} = \sigma_T = k_1$, that is :

$$\sigma_{z1} - p(z_1) \cdot \frac{\eta_1}{s_1} = k_1 \tag{7}$$

By equalizing values $F_I = F_{su}$ of stress $p(z_I)$ and using the condition of flow and condition of equilibrium, we derive dependence of length z_I (initial forming of axial instability as the function of deformation force, hardening coefficient k_I , axial stress σ_{zI} and wall thickness s_I) in the following form:

$$(z_1 - dz_1) \cdot dz_1 = \frac{F_{su} \cdot \lambda}{\pi(\sigma_{z1} - k_1) \cdot s_1} \tag{8}$$

The stress state is defined assuming that $(z_I - dz_I) dz_I > 0$, that is, $dz_I > 0$ and $dz_I < z_I$.

The length of the free upsetting zone, i.e. the zone of contactless forming which occurs at a certain stage of the necking process, is an important factor in solving the problem of stabile forming of workpieces with similar dimensions.

At the end of the cylindrical section of the billet, there begins a contactless bending for the angle α with radius R_2 , which is the function of wall thickness and billet diameter. The zone of contactless forming (2) (see Fig. 2.) occurs at a certain distance from the beginning of the contact necking, and its length depends primarily on mechanical properties of workpiece material. Disregarding the contactless zone can lead to errors in calculation of necking stress. The length of the contactless zone is determined between points of contact with spherical tool and the transition into cylindrical section of the tube (Fig. 2).



Figure 4 - Stress state in the contactless zone

From the stress equilibrium equation in the contactless zone in the direction of the meridian plain which is normal to the bending radius (Fig. 4), there follows:

$$(\sigma_{z2} + d\sigma_{z2}) \cdot r_2 d\varphi_2 s_2 - \sigma_{z2} (r_2 - dr_2) d\varphi_2 \cdot s_2 - 2\sigma_{\varphi_2} \sin \frac{d\gamma_2}{2} R_2 d\alpha_2 \cdot s_2 = 0$$
(9)

Introducing substitutions in the above equation:

$$\sin\frac{d\gamma_2}{2} = \frac{r_2 d\varphi_2 - (r_2 - dr_2) d\varphi_2}{2R_2 \cdot d\alpha_2}$$
(10)

and disregarding higher order members, we derive the stress state equation:

$$d\sigma_{z2} \cdot r_2 + \sigma_{z2}dr_2 - \sigma_{\omega 2}dr_2 = 0 \tag{11}$$

which can be written as:

$$\frac{d\sigma_{z2}}{dr_2} + \frac{\sigma_{z2} - \sigma_{\varphi 2}}{r_2} = 0 \tag{12}$$

This equation satisfies the condition of plastic flow according to criterion of highest shear stress for volumetric load (Tresca-Saint Venant criterion) according to which plastic forming occurs when the difference between maximum and minimum principal normal stress reaches the flow

boundary $\sigma_{z2} - \sigma_{\varphi 2} = k_2$. We arrive at the following equation of stress state: $\frac{d\sigma_{z2}}{dr_2} - \frac{k_2}{r_2} = 0$ which

represents the Cauchy problem, where r_2 is an independent variable, σ_{z2} is dependent variable, $d\sigma_{z2}/dr_2$ is a derivative of a dependent variable with respect to an independent variable, k_2 is a constant (deformation strengthening) at the deformation zone boundary and (σ_{z1} , R_1) are the values which should be contained in the particular solution of the equation. General solution is:

$$\sigma_{z2} = k_2 \cdot \ln r_2 + \ln C \tag{13}$$

that is $\sigma_{z2} = ln C \cdot r_2^{k_2}$ where *C* is a free constant calculated from $\sigma_{z2}(r_2 = R_1) = \sigma_{z1}$. The solution of the Cauchy problem is:

$$\sigma_{z2(r_2)} = \sigma_{z1} - \ln\left(\frac{R_1}{r_2}\right)^{k_2} \tag{14}$$

Upon inserting numerical values $R_1 = 16 \text{ mm}$, $r_2 = 15, 32 \text{ mm}$, $k_2 = 495 \text{ MPa}$ we derive:

$$\sigma_{z2(R_2)} = -374, 20 - ln \left(\frac{16}{15, 32}\right)^{495} = -395, 7 \, Mpa \tag{15}$$

After contactless bending there exists a zone of spherical necking in which appears an additional pressure p_3 of the necking ring at the outer side of the workpiece, creating a component of normal stress $p_3 = \sigma_{\rho 3}$ (Fig. 5), which in combination with friction coefficient μ results in stress τ_3 on the contact surface. From the equation of equilibrium of stresses in the meridian plain, in the tangential direction we have:

 $\left(\sigma_{z3} + d\sigma_{z3}\right) \cdot r_3 d\varphi_3 \cdot s_3 \cdot \cos \frac{d\varphi_3}{2} - \sigma_{z3} \left(r_3 - dr_3\right) d\varphi_3 \cdot s_3 \cdot \cos \frac{d\varphi_3}{2} - \mu \cdot \sigma_{\rho3} \cdot r_3 d\varphi_3 \cdot r_3 d\varphi_3 - 2\sigma_{\varphi3} r_3 d\varphi_3 \cdot s_3 \cdot \sin \frac{d\varphi_3}{2} = 0$ (16)



Figure 5 - Stress state of volume element in spherical necking zone



Figure 6 - Stress state of volume element in bending zone across the fillet radius r

If, in the above equation, we make a substitution:

$$\cos\frac{d\varphi_3}{2} \cong 1; \quad \sin\frac{d\varphi_3}{2} = \frac{dr_3 \cdot d\varphi_3}{2r_3 \cdot d\varphi_3} \cong \frac{d\varphi_3}{2} = \frac{dr_3}{2r_3}$$

and disregard higher order members, there follows

$$d\sigma_{z3} \cdot r_3 \cdot s_3 + \sigma_{z3} \cdot dr_3 \cdot s_3 - \mu \cdot \sigma_{\rho 3} \cdot r_3 \cdot dr_3 - \sigma_{\varphi 3} \cdot s_3 \cdot dr_3 = 0$$

$$\tag{17}$$

if, according to maximum shear stress hypothesis $\sigma_{z3} - \sigma_{\varphi 3} = k_3$, then for the necking in the deformation zone 3, there follows the equation of stress state in the following form:

$$\frac{d\sigma_{z3}}{dr_3} - \sigma_{z3} \cdot \frac{\mu}{r_3} + k_3 \cdot \frac{1}{r_3} \cdot (\mu + 1) = 0$$
(18)

This is a first-order linear differential equation whose general solution is:

$$\sigma_{z3} = C \cdot r_3^{\ \mu} - \frac{k_3}{\mu} \tag{19}$$

where (C) is a free constant which can be found from the initial conditions. The solution of the Couchy problem is:

$$\sigma_{z3(r_3)} = \frac{\mu \cdot \sigma_{z2} + k_3^{\mu}}{R_2^{\mu} \cdot \mu} \cdot r_3^{\mu} - \frac{k_3}{\mu}$$
(20)

Values $\mu = 0,095$, $R_2 = 15, 32$ mm, $r_3 = 13,75$ mm and $k_3 = 503$ MPa define dimensions and mean value of hardening on the section of the deformation zone to which the equation applies. When these values are fed into the solution of the Cauchy problem, we derive values of axial stress: $\sigma_{z3} = -447.1$ MPa

Stresses which occur due to bending and friction resistance (Fig.6) are determined from the condition of equilibrium along tangent in the meridian plane:

$$(\sigma_{z4} + d\sigma_{z4}) \cdot r_4 \cdot d\varphi_4 \cdot s_4 - \sigma_{z4}(r_4 - dr_4) \cdot d\varphi_4 \cdot s_4 - \mu \cdot p_4 \cdot r_4 \cdot d\varphi_4 \cdot r \cdot d\alpha_4 - 2\sigma_{\varphi 4} \cdot r \cdot d\alpha_4 \cdot s_4 \cdot sin \frac{d\gamma_4}{2} = 0$$

$$(21)$$

If, in the above equation, we disregard higher order members and suppose that:

$$d\alpha_4 \cong \frac{dr_4}{r} \quad \text{and} \quad \sin\frac{d\gamma_4}{2} = \frac{r_4 \cdot d\varphi_4 - (r_4 - dr_4) \cdot d\varphi_4}{2rd\alpha_4} = \frac{dr_4 \cdot d\varphi_4}{2rd\alpha_4} \tag{22}$$

while $p_4 = \sigma_{\rho 4}$, there follows equation of stress state in the following form:

$$d\sigma_{z4} \cdot r_4 \cdot s_4 + \sigma_{z4} \cdot dr_4 \cdot s_4 - \mu(\sigma_{z4} - k_4) \cdot r_4 \cdot dr_4 - (\sigma_{z4} - k_4) \cdot s_4 \cdot dr_4 = 0$$

$$(23)$$

From the condition of equilibrium in radial direction:

$$p_4 \cdot r_4 \cdot d\varphi_4 \cdot r \cdot d\alpha_4 - 2\sigma_{\varphi 4} \cdot r \cdot d\alpha_4 \cdot s_4 \cdot \sin \frac{d\gamma_4}{2} = 0$$
(24)

we derive dependence between radial and circular stresses in the following form:

$$p_4 \cdot r_4 - \sigma_{\varphi 4} \cdot s_4 = 0; \quad p_4 = \sigma_{\varphi 4} \cdot \frac{s_4}{r_4}$$
 (25)

This equation satisfies condition of plastic flow according to criterion of maximum shear stress for a volumetric load (Tresca-Saint Venant criterion) according to which plastic forming occurs when the difference between maximum and minimum principal normal stress reaches the flow stresses $\sigma_{z4} - \sigma_{\varphi 4} = k_4$.

Rearanging gives: $\frac{d\sigma_{z4}}{dr_4} - \sigma_{z4} \cdot \frac{\mu}{r_4} + k_4 \cdot \frac{1}{r_4} \cdot (\mu+1) = 0$. This is a first-order linear differential equation with following solution:

equation with following solution:

$$\sigma_{z4(r_4)} = \frac{\mu \cdot \sigma_{z3} + k_4}{R_3^{\mu} \cdot \mu} \cdot r_4^{\mu} - \frac{k_4}{\mu}$$
(26)

Substituting $\mu = 0,095$, $R_3 = 13,75$ mm, $r_4 = 12, 7$ mm and $k_4 = 516$ MPa, which define dimensions and mean value of hardening on the section of the deformation zone to which the equation applies, we derive axial stress σ_{z4} = - 482,6 Mpa.

Due to the size of the cone angle, the deformation of material in the conic section of the necking ring is not stationary by nature. This accounts for the increase in necking force due to higher stress. Stresses p_5 on the contact surface (Fig. 7) generates friction force $\tau_5 = \mu p_5$.



Figure 7 - Stress state of volume element in the zone of conical necking

Equilibrium of stress state for a volume element in tangent direction in the meridian plane is given by following equation:

$$\left(\sigma_{z5} + d\sigma_{z5}\right) \cdot r_5 \, d\varphi_5 \cdot s_5 - \sigma_{z5} \left(r_5 - dr_5\right) d\varphi_5 \cdot s_5$$

$$-\mu \cdot p_5 \cdot \left(r_5 + \frac{s_5}{2}\right) d\varphi_5 \cdot \frac{dz_5}{\cos 1^\circ} - 2\sigma_{\varphi 5} dz_5 \cdot s_5 \cdot \sin \frac{d\beta}{2} = 0$$
(27)

Supposing geometric dependency in a volume element:

$$\sin\frac{d\beta}{2} \approx \frac{d\beta}{2} = \frac{dr_5 \cdot d\varphi_5}{2dz_5}; \quad \frac{dz_5}{\cos 1^\circ} = \frac{dr_5}{\sin 1^\circ}.$$

According to the hypothesis of maximum shear stress, $\sigma_{z5} - \sigma_{\varphi 5} = k_5$, so for the case of necking in the deformation zone (5) we derive following equation of stress state:

$$d\sigma_{z5} \cdot r_5 \cdot s_5 + \sigma_{z5} \cdot dr_5 \cdot s_5 - (\sigma_{z5} - k_5) \cdot (\mu \cdot \frac{r_5}{\sin 1^\circ} + s_5) \cdot dr_5 = 0$$
(28)

After rearrangement:

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$$\frac{d\sigma_{z5}}{dr_5} - \sigma_{z5} \cdot \frac{1}{r_5} \cdot \frac{\mu}{\sin^0} + k_5 \cdot \frac{1}{r_5} (\frac{\mu}{\sin^0} + 1) = 0$$
(29)

This is a first-order linear differential equation with following solution:

$$\sigma_{z5(r_5)} = \frac{\mu \cdot \sigma_{z4} + k_5}{R_4^{\mu} \cdot \mu} \cdot r_5^{\mu} - \frac{k_5}{\mu}$$
(30)

Substituting values $\mu = 0,095$, $R_4 = 12, 7$ mm, $r_5 = 12, 25$ mm and $k_5 = 521$ MPa, which pertain two dimensions and a mean value of hardening factor in the section of the deformation zone which is defined by the differential equation, we derive maximum stress: $\sigma_{z5} = -515,1$ MPa

The magnitude of radial stresses along deformation zone depends on the wall width/diameter ratio s_0/D_0 . From the equation of stress equilibrium in radial direction:

$$p_5 \cdot r_5 \cdot d\varphi_5 \cdot dz_5 - 2\sigma_{\varphi 5} \cdot \sin \frac{d\varphi_5}{2} \cdot s_5 \cdot dz_5 = 0$$
(31)

we derive dependences between radial and circular stresses in the form of: $p_5 = \sigma_{\varphi 5} \cdot \frac{s_5}{r_5}$.

Compared to axial and circular stresses, the magnitude of radial stresses is lower and depends on the wall thickness and billet diameter, so radial stresses can be disregarded when the tube is drawn through the necking ring [3]. This ratio between radial and circular stresses is proportional to wall thickness and radius (or diameter, in certain phases) (s_i / r_i), and it ranges between (10÷30)% from zone (1) to zone (5). This must be taken into consideration during stress state analysis, bearing in mind the sensitivity of the process to instability of forming. Fig. 8 shows the stress diagram obtained by solving differential equations of equilibrium state in characteristic zones.



Figure 8 - Approximation of stresses obtained by solving differential equations along the deformation zone

3. COMPARISON OF RESULTS OBTAINED BY SOLVING DIFFERENTIAL EQUATIONS AND FEM SIMULATION

The numerical simulation of the necking of seamless steel pipe billets was conducted in CAMPform 2D software package, dedicated to FEM analysis of 2D processes. The software was develpoed at the KAIST Institute (Korea Advanced Institute of Science and Technology), Taejon. Software packages CAMPform 2D and 3D are made for simulation of bulk forming processes (forging, cold and warm extrusion, rolling...). The processor is based on thermo-solid-viscoplastic finite elements which were developed by Kobayashi et al. [4], [5], [6]. This approach in fact incorporates methods for solving equilibrium equations and the energy equation, based on the solid-viscoplastic constituent model with von Mises flow criterion. Finally, it would be useful to compare deformation and stress fields obtained by FEM analysis with the analysis results obtained in this paper, by solving differential equations.

Given in Fig. 9 is the stress diagram along the meridian cross-section with a predominantly upsetting stress state caused by the axial and circular stresses in the course of the necking process. This justifies the use of CAMPform numerical simulation to model the forming of axisymmetrical tubular billets of similar dimensions.



Figure 9 - Stress diagram of equilibrium state in the meridian section, obtained by FEM simulation

4. CONCLUSION

The comparison between the results obtained by FEM simulation of the process, i.e. the stress fields over the entire deformation zone of the tubular billet, shows satisfactory compliance with the stress fields obtained by solving the equilibrium differential equations in the characteristic zones.

Considering the results obtained by experiments on an example, which has broad practical application, the solutions obtained by numerical FEM model can be of practical assistance in solving similar problems, such as necking and expanding or a combination of the two used in the forming of tubular billets.

5. REFERENCES:

- [1.] B. Mišić, M. Šljivić: Production development of one axial- symmetrical work-piece out of seam tube through necking-in method. Journal for Technology of Plasticity, Vol.26(2001), Number 2, pp.27-33.
- [2.] B. Mišić: Stabilnost procesa sužavanja izradaka iz čeličnih šavnih cijevi, Doktorska disertacija, Mašinski fakultet u Banjoj Luci, 2004.
- [3.] A.Mock: Beitrag zum Draht-und Rohrziehen mit ultraschallerregter Matrize, Institut fur Umformtechnik Universitat Stuttgart, 2000.
- [4.] Y.T.Im: A computer-aided-design system for forming processes, Jour. of Mater. Proc. Tech, 89-90 (1999), pp. 1-7.
- [5.] S.I.Oh, W.T. Wu, J.P.Tang, A.Vendhanayagam: Capabilities and applications of FEM code DEFORM: the perspective of the developer, Jour. of Mater. Proc. Tech, 27 (1991), pp. 25-42.
- [6.] V. Mandić, Fizičko modeliranje i numerička simulacija kao osnova novog koncepta projektovanja alata za toplu zapreminsku obradu, Doktorska disertacija, Mašinski fakultet u Kragujevcu, 2002.

IDENTIFIKACIJA NAPONSKOG STANJA KOD OBRADE **CLIEVNIH IZRADAKA**

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REZIME

Analiza naprezanja, odnosno naponskog stanja je prikazana na primjeru obrade deformisanjem cijevnog izratka koji je predmet eksperimentalnih istraživanja iz razloga provjere pravilnosti teorijske postavke kao i identifikacije osnovnih parametara procesa. Za stabilnost procesa oblikovanja izratka često je neophodno izvršiti analizu naponsko deformacionog stanja duž zone deformisanja. Veličinu napona tečenja je moguće odrediti eksperimentalno ili rješavanjem jednačina naponskog stanja elemenata volumena na karakterističnim mjestima deformacione zone.

U radu je izvršeno određivanje naponskog stanja kod obrade cijevnih izradaka uz korišćenje diferencijalnih jednačina ravnoteže elementa volumena i uslova plastičnog tečenja u okvirima ograničenja odnosa normalnih i tangencijalnih napona na vanjskoj odnosno unutrašnjoj površini. Izvedena eksperimentalna istraživanja na deformisanju cijevnih izradaka su u potpunosti potvrđena (FEM) numeričkom simulacijom korišćenjem programskog paketa CAMPform 2D u Windows okruženju.

KLjučne reči: naponsko stanje, Cevasti pripremci, Diferencijalne jednačine, FEM simulacija