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EXTRUSION PARAMETERS DETERMINATION BASED ON DIFFERENT OPTIMIZATION APPROACHES

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ABSTRACT

Purpose of this research is determination of the optimal cold forward extrusion parameters with objective the minimization of tool load. This paper deals with the different optimization approaches relating to determine optimal values of logarithmic strain, die angle and coefficient of friction with the purpose to find minimal tool loading obtained by cold forward extrusion process. To achieve this, it has been carried out two experimental plans based on factorial design of experiment and orthogonal array. By using these plans it was performed classical optimization, according to response model of extrusion forming force, and the Taguchi approach, respectively. Experimental verification of optimal forming parameters with their influences on the forming forces was done. The experimental results showed an improvement in minimization of tool loading. It was compared results of optimal forming parameters obtained with different *optimization approaches and based on that the analysis of the characteristics (features and limitations) of both techniques.*

1. INTRODUCTION

The metal forming process is characterized by various process parameters including the shape of the workpiece and product, forming sequence, shapes of tools or dies, friction, forming speed, temperature and material property of the workpiece and those of the tools. Therefore, determination of the optimal forming parameters by using optimization techniques is continuous engineering task with main aim to reduce the production cost and achieve desired

product quality [1,2]. Forming technologies, that have been applied for a number of years in a definite conventional form, can be innovated by applying knowledge from the area of modelling, simulations, optimizations, theory of processes, computer technique and artificial intelligence [3].

The optimization methods have been improved by development of applied mathematics, statistics, operational researches, design of experiment, simulation and information-computational methods. Today, there are more different optimization methods. The use of the existing methods depend on objects modelling, required degree of model accuracy, type of process and necessity of optimization.

In this research work, mathematical modelling of the extrusion force and the different optimization approaches relating to determine optimal values of logarithmic strain, die angle and coefficient of friction with the purpose to find optimal tool load obtained by cold forward extrusion process [4,5,6].

Hence, optimization i.e. minimization of the cold forward extrusion force has been carried out by two experimental plans based on factorial design of experiment and orthogonal array. By using these plans it was performed classical mathematical optimization, according to response model of extrusion forming force, and the Taguchi approach, respectively. Finally, the confirmation experiment was conducted to verify the optimal extrusion parameters with the minimal tool load and to confirm the effectiveness of these approaches. The value of presented techniques and obtained results have a practical implication on the smallest energy consumption, longer tool life, better formability of the work material and the quality of the finished product.

2. EXPERIMENTAL PROCEDURE AND RESULTS

The processes of cold and hot extrusion are classified depending upon the direction of material flow in relation to the tool movement direction. Another method of classifying these processes is by their geometry, namely, solid and hollow components [7].

In the solid forward extrusion process, analyzed in this paper, the flow of metal is in the same direction as the direction of action of the machine (punch), where final product is a solid workpiece with a profile determined by the shape of the die opening, shows in Fig. 1.

Fig. 1. Extrusion die geometry with initial and formed part

Forward extrusion force value can be obtained both experimentally using definite measurement equipment and analytically according to well know expression for the total extrusion force [3,6,7]. Consequently, it can be concluded that forward extrusion force basically depends on material properties, logarithmic strain, die angle, coefficient of friction and initial geometry of workpiece (billet).

From that point of view, the experiment has been carried out by using central composition design with five levels of the three main independent parameters, namely, logarithmic strain (*φ*), die angle (α) and friction coefficient (μ) (Table 1.) [3,4,5,6].

Table 1. Levels of independent extrusion parameters

Tuole 1, Develop yr maepenaeni emir ilolon parametero								
Symbol	Parameters / Levels	Lowest	Low	Centre	High	Highest		
	Coding	-1.6817	- 1		$^+$	$+1.6817$		
A	Logarithmic strain φ	0.112	0.308	0.596	0.884	1.080		
	Half-die angle α (°)		18	30	42	50		
	Coefficient of fricition μ	0.066	0.08	0.10	0.12	0.134		

Overall the number of experiments which was conducted for this central composition design is $N = 2³ + 6 + 6 = 20$ trials. There are eight ($2³$) factorial design with added six star points and center point repeated six times to calculate pure error.

2.1. Experiment setup

The forward extrusion operations were performed on hydraulic press with alloyed carbon steel, according to DIN 16MnCr5, (workpiece material) as rod billet (Table 2.). Experiments were run with different friction conditions what for used the following lubricants: $MoS₂$, phosphate surface & oil, grease, oil, moist oil with five coefficient of frictions according to level parameters, respectively. Initial diameter of workpiece ($d_0 = 30$ mm) and height ($h_0 = 37$ mm) for all the experiments are constant.

Mechanical properties of steel 16MnCr5									
tensile strength (N/mm^2) yield strength (N/mm^2)		Brinell hardness $_{\rm HB}$	elongation $\%$ reduction $\%$						
570	400 160		26	65					
Chemical composition %									
Si C Mn Сr									
0.16	0.30	1.15	0.95	0.030					

Table 2. Mechanical properties and chemical composition of steel 16MnCr5 (DIN)

Based on compression test obtained data for flow stress curve according to Hollomon [7] has the following form:

$$
\sigma_f = C \varphi^n = 960 \varphi^{0.18}
$$

where is:

 σ_f – flow stress, *C* – constant, *n* – strain-hardening coefficient.

3. EXTRUSION FORCE MODEL PREDICTION

Design of experiment is a powerful tool for modelling and analysing the influence of process parameters. On the basic of performed experiment can be represented the functional relationship between response of extrusion process, in this case the extrusion force, and the investigated independent parameters by the following polynomial form of mathematical model [3,4,5,6,8,9]:

$$
Y = b_0 + \sum_{i=0}^{k} b_i X_i + \sum_{1 \le i \le m}^{k} b_{im} X_i X_m + \sum_{i=0}^{k} b_i X_i^2 + \sum_{1 \le i \le m \le k} b_i X_i X_m X_k + \varepsilon
$$
\n(1)

Today, different kind of software tools for design of experiment have been developed. In this paper, the MS Excel® package was used to calculate the all coefficients values including interactions (Table 3).

Table 3. Values of coefficients obtained by MS Excel®

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper $95%$
Intercept	607.642112	8.486322	71.60253	$1.021E-13$	588.4447	626.8395
b ₁	170.391547	5.630156	30.26409	2.296E-10	157.6552	183.1279
b ₂	13.7988415	5.630156	2.450881	0.036703	1.062534	26.53515
b ₃	48.9601595	5.630156	8.696057	1.129E-05	36.22385	61.69647
b_{11}	12.2010429	5.480255	2.226364	0.053010	-0.196165	24.59825
b_{22}	51.6142631	5.480255	9.418222	5.879E-06	39.21705	64.01147
b_{33}	6.19185237	5.480255	1.129847	0.287752	-6.205356	18.58906
b_{12}	-1.75	7.356529	-0.237884	0.817298	-18.39164	14.89164
b_{13}	0.25	7.356529	0.033983	0.973632	-16.39164	16.89164
b_{23}	4.75	7.356529	0.645685	0.534597	-11.89164	21.39164
b_{123}	11.5	7.356529	1.563237	0.152433	-5.141637	28.14164

After taking into consideration only significance coefficients (highlight) the obtained mathematical model in coding form is:

$$
Y = 607.64 + 170.39 X1 + 13.79 X2 + 48.96 X3 + 12.2 X12 + 51.61 X22 + 11.5 X1 X2 X3
$$
 (2)

or after transformation Eq. (2) extrusion force model as function of the logarithmic strain (φ) , die angle (α) and friction coefficient (μ) has the following physical form:

$$
F = 52.93 + 915.33\varphi + 147\varphi^2 - 10.45\alpha + 0.355\alpha^2 + 5423.62\mu
$$

- 16.56\varphi\alpha - 4991\varphi\mu - 98.325\alpha\mu + 165.6\varphi\alpha\mu. (3)

For the 95% confidence level the $R^2 = 0.99$ what shows a good interdependency of the input parameters (φ, α, μ) and response (F) . In Table 4. is presented results obtained by predicted model (3) and analytical model (6) and it was compared with experimental results. According to that can be concluded, that forming force model (3) decribes accurately enough (model explains 99% of the variability in force F) the experimental results within experiment domain.

Table 4. Design of experiments with experimental and model results									
Parameters					Extrusion force F (kN)				
No $\varphi \leftrightarrow X_1$			$\alpha \leftrightarrow X_2$		$\mu \leftrightarrow X_3$		Experiment	Predicted	Analytical
trial	logarithmic strain	coding	half-die $angleo$)	coding	friction coefficient	coding	$(F_{average})$	model (3)	model (6)
	0.308	-1	18	-1	0.08	-1	445	426.81	332.05
2	0.884	$+1$	18	-1	0.08	-1	790	790.59	677.40
3	0.308	-1	42	$+1$	0.08	-1	478	477.40	433.92
4	0.884	$+1$	42	$+1$	0.08	-1	770	795.19	762.98
5	0.308	-1	18	-1	0.12	$+1$	560	547.73	390.48
6	0.884	$^{+1}$	18	-1	0.12	$+1$	860	865.51	757.39
	0.308	-1	42	$+1$	0.12	$+1$	566	552.32	485.22
8	0.884	$+1$	42	$+1$	0.12	$+1$	905	916.11	819.68
9	0.596	$\mathbf{0}$	30	θ	0.10	0	610	607.64	564.60
10	0.596	Ω	30	0	0.10	0	614	607.64	564.60
11	0.596	0	30	θ	0.10	0	605	607.64	564.60
12	0.596	Ω	30	0	0.10	0	611	607.64	564.60
13	0.596	Ω	30	0	0.10	0	606	607.64	564.60
14	0.596	θ	30	0	0.10		597	607.64	564.60
15	0.112	-1.6817	30	0	0.10	0	338	355.56	304.95
16	1.080	1.6817	30	Ω	0.10	0	963	928.76	862.10
17	0.596	Ω	10	-1.6817	0.10	0	725	730.45	554.76
18	0.596	0	50	1.6817	0.10	0	799	776.87	661.95
19	0.596	0	30	θ	0.066	-1.6817	556	525.29	517.39
20	0.596	0	30	θ	0.134	1.6817	711	689.99	611.81

Table 4. Design of experiments with experimental and model results

In this particular case it was proposed second-order model with interactions from the two main reasons:

(i) it is not possible to use classical (mathematical) optimization at the first-order model,

(ii) better explain the behaviour of the process parameters.

4. OPTIMIZATION OF EXTRUSION FORCE

Determination of the optimal forming parameters by using optimization techniques is continuous engineering task with main aim to reduce the production cost and achieve desired product quality. For a forming process such as forward extrusion, the forming conditions play an important role in the efficient use of a machine tool. Since the cost of extrusion process is sensitive to the forming conditions optimum values have to be determined before a part is put into production.

To select the forming parameters properly, there are considerable number of optimization techiques and some of them are shown in Fig. 2 [6,10,11]. The optimum forming parameters, in this case will be determined by the different optimization approaches, classical mathematical based on experimental obtained model, analytical based on literature known equation and Taguchi, with the objective to minimize forward extrusion force. It has already known that minimal extrusion force is possible to achieve with low strain and coefficient of friction as technological and tribological parameters, respectively. In this case study emphasis is on geometrical aspect of the extrusion force minimization, that is, it will be a function of die angle only.

4.1. Classical mathematical optimization

In classical mathematical analysis the optimization of extrusion process parameters were carried out by derivation of the obtained mathematical model (3). In this paricular case derivation of predicted mathematical model will be performed with the aim to find optimal half-die angle (α) :

$$
\frac{dF}{dX_i} = 0 \qquad i=1,2,3, \text{ that is, for die angle} \qquad \frac{dF}{d\alpha} = 0 \tag{4}
$$

in this case study:

$$
\frac{dF}{d\alpha} = 0 \Rightarrow -10.45 + 0.71\alpha - 16.56\varphi - 98.325\mu + 165.6\varphi \mu
$$

or optimal half-die angle (^α*opt*) (Table 5.) is:

$$
\alpha_{opt} = \frac{10.45 + 16.56\varphi + 98.325\mu - 165.6\varphi\,\mu}{0.71}
$$
\n
$$
(5)
$$

Furthermore, based on literature known mathematical model for total solid forward extrusion force [7]:

$$
F_{\text{tot}} = A_0 \cdot \sigma_{f,\text{m}} \left[\frac{2}{3} \hat{\alpha} + \left(I + \frac{2\mu}{\sin 2\alpha} \right) \varphi_{\text{max}} \right] + \pi \cdot d_0 \cdot l \cdot \mu \cdot \sigma_{f,0} \tag{6}
$$

or for minimum force requirements optimal die angle is calculated by the following equation:

$$
\frac{dF_{tot}}{d\alpha} = 0 \implies \cos 2\alpha_{opt} = -3\mu \varphi_{max} \pm \sqrt{9\mu^2 \varphi_{max}^2 + I} \tag{7}
$$

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It can further be seen from (7) that the optimum die opening angle is dependent on the coefficient of friction (μ) and the logarithmic strain (φ) but independent of the material properties [7].

4.2. Taguchi approach

according the classical plan.

In this paper optimization based on Taguchi approach [12,13,14,15] is used to achieve the more efficiency extrusion parameters, especially for die angle, and to compare results obtained with both techniques. Table 6 shows that the experimental plan has three levels and an appropriate Taguchi orthogonal array with notation L_9 (3⁴) was chosen (Table 7.).

Table 6. Levels of independent extrusion parameters according to Taguchi approach Symbol Parameters Level 1 Level 2 Level 3 Degrees of freedom (DOF) A logarithmic strain, *φ* 0.308 0.596 0.884 2
B half-die angle, *α* (^o) 18 30 42 2

B half - die angle, *α* (°) 18 30 42 2
C coefficient of friction *μ* 0.08 0.10 0.12 2

C coefficient of friction, μ 0.08 0.10 0.12

Central composition plan (Table 4.)	-1		
The last column of parameters notation with D (Table 7.) was used to estimate the experiment			
error. The right side of the table includes the average results (each trial has 3 samples) of the			
measured force and the calculated signal-to-noise (S/N) ratio with associated the trial number			

Table 7. Three-level orthogonal array, L₉ (3⁴), with experimenatal results (average) and *calculated signal-to-noise (S/N) ratios*

The *S/N* ratio, as the yardstick for analysis of experimental results, is calculated according to the following equation:

$$
S/N = \eta = -10\log_{10}\left(\frac{1}{n}\sum_{i=1}^{n}y_i^2\right)
$$
 (8)

where is:

^η – signal-to-noise ratio (*S/N*)

n – number of repetitions of the experiment

 y_i – measured value of qualtiy characteristic

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The above equation, which is used to calculate the *S/N* ratio, is in relation to the *smaller-isbetter* quality characteristics, what in the particular case means minimization of extrusion force. Taguchi uses the *S/N* ratio to measure the quality characteristic deviating from the desired value.

The influence of each control parameter can be more clearly presented with response graphs (Fig. 3.). *S/N* ratios for all control parameters are calculated based on the following equations:

$$
AI = \frac{1}{3}(\eta_1 + \eta_2 + \eta_3) = \frac{1}{3}(-52.968 - 53.065 - 55.061) = -53.698
$$

\n
$$
A2 = \frac{1}{3}(\eta_4 + \eta_5 + \eta_6) = \frac{1}{3}(-56.365 - 56.445 - 56.192) = -56.334
$$

\n
$$
A3 = \frac{1}{3}(\eta_7 + \eta_8 + \eta_9) = \frac{1}{3}(-58.691 - 57.386 - 58.434) = -58.17
$$

According to above equations, on the same manner, are calculated *S/N* ratios for parameter B(B1,B2,B3) & C(C1,C2,C3).

Fig. 3. S/N graphs for control parameters

A response graph shows the change of the *S/N* ratio when control parameter is changed from one level to the other. The slope of the line determines the power of the control parameters influence what is presented in Table 8 as contribution. The settings of control parameters for achievement of the best value of the quality characteristic can be determined by response graphs presented. Best value is at the higher value of the *S/N* ratio, or according to Fig. 3., it can be concluded that the minimal extrusion force will be achieved at following level combination of parameters (A1B2C1) (Table 9):

- logarithmic strain φ = 0.308,
- die angle $\alpha = 30^{\circ}$ and
- Fiction coefficient μ = 0.08.

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5. RESULTS ANALYSIS

The optimal parameter values for the different approaches are presented in Table 9. The presented optimization techniques give accurate results (confirmation test) with small deviation between each other, except the analytical method. Final step is to verify the improvement using optimal level of parameters (about 10%). Since, the model (3) has the interaction parts the optimal die angle depends on strain and friction, i.e. it has been established optimal die angle path (Fig. 4) for the different input conditions.

Table 9. The comparision of the optimal results and confirmation test

	Initial	Optimal forming parameters			Confirmation test
parameters		Prediction model (3)	Taguchi approach	Analytical model (7)	
Level	A1B1C1	A 1 C 1			A1C1
Force F (kN)	445	396.41	432.11	318.01	402
Optimal half-die angle α (°)		$B = \alpha = 27.23^{\circ}$	$B2 = \alpha = 30^{\circ}$	$B = \alpha = 10.88^\circ$	$\alpha_{\rm out} = 27.23^{\circ}$

Fig. 4. The optimal die angle path

6. CONCLUSIONS

In this paper is shown an application of the different optimization approaches to find optimal cold forward extrusion parameters with emphasis on geometrical aspect of the process, that is, die angle. The both presented optimization techniques, classical and Taguchi, have its features, merits and limitations what is presented on the practical case with the following conclusions are made:

(i) Classical experimental design methods are too complex and not easy to use. A large number of experiments have to be carried out especially when the number of process parameters increases. To solve this problem, the Taguchi approach uses a special design of orthogonal arrays to study the entire parameter space with a small number of experiments, what is obviously if we compare Table 4 and Table 7. Furthermore, to obtain optimal value of process parameters the classical method needs the prediction model which was used for optimization procedure, what is not necessary for orthogonal arrays design. Also, the parameters value needs to be defined strictly numerical not as description of state.

- (ii) On the other hand, advantage of clasical experimental design methods are possible to obtain mathematical model which is powerful tool to predict response for any of input parameters value within the experiment domain, and optimal values can to be any of parameters point i.e. parameters are continuous and can take any real value. This is impossible in Taguchi approach, because optimal value have to be one of parameter levels, see Table 9, and solution may give a value of the objective functions that is very far from the original optimum value. In addition, Taguchi approach is better for parameters can only have discrete values in contrast to classical optimization technique and continuous values.
- (iii) Finally, the both optimization techniques presented here have potentiality (more or less) to improve initial process parameters or in study case minimization of extrusion force by means of the optimal die angle with high accuracy what is also verified by confirmation experiment.

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ODREĐIVANJE PARAMETARA ISTISKIVANJA NA TEMELJU RAZLIČITIH OPTIMIZACIJSKIH PRISTUPA

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REZIME

Svrha istraživanja predstavljenog u ovom radu je bila određivanje optimalnih parametara procesa istosmjernog istiskivanja s ciljem minimizacije opterećenja alata. U ostvarenju toga cilja pristupilo se različitim poznatim optimizacijskim metodama kako bi se došlo do optimalnih vrijednosti pojedinih ulaznih parametara procesa istiskivanja, kao što su logaritamska deformacija, ugao matrice i koeficijent trenja. Dobiveni eksperimentalni rezultati bazirani su na dva različita pristupa-plana od kojih je jedan centralni kompozicijski plan (20 eksperimenata) a drugi je poznat kao ortogonalni niz (9 eksperimenata). Kod prvog plana koristeći dobivene rezultate pristupilo se modeliranju sile istiskivanja a na temelju tako dobivenog modela i primjene klasične-matematičke optimizacije dobiveni su optimalni parameteri procesa.

Drugi plan baziran na Taguchi pristupu koristi manji broj eksperimenta i nema potrebe za modeliranjem sile procesa a uz korištenje odgovarajuće statističke analize i definiranog uvjeta «manje je bolje» daje optimalne vrijednosti parametara koje u prezentiranom slučaju relativno malo odstupaju od klasične metode optimiranja. Također izvršena je usporedba dobivenog optimalnog rješenja na temelju analitičke jednačine poznate iz literature i pokazalo je se da tako dobiveno rješenje značajno odstupa. Na kraju je provedena i eksperimentalna verifikacija dobivenog optimalnog rješenja koja je pokazala visoku korelaciju sa matematički dobivenim rješenjem i smanjenje opterećenja alata za 10%. Također je prikazan i utjecaj (%) svakog pojedinog ulaznog parametra na silu procesa istiskivanja. Definirane su osnovne prednosti i mane prezentiranih tehnika optimiranja a samim time i moguća područja primjene navedenih optimizacijskih metoda.

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