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MODELLING OF FORWARD EXTRUSION PROCESS FOR HOLLOW ELEMENTS ON BASE OF NONLINEAR ADAPTIVE FINITE ELEMENT METHOD

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ABSTRACT

Adaptive strategies are nowadays considered a standard tool in practical finite element computations. For any problem, adaptivity is an essential tool to obtain numerical solutions with a controlled accuracy. This is tool, which offer solutions that follow disruption within continuum structure with possibility of handle process and stress – strain analysis. For some problems typically in the nonlinear domain adaptive strategies are even more fundamental, without them a finite element solution simply cannot be computed. This is the case, for instance, with problems in nonlinear solid mechanics applied on extrusion technology of aluminium in die area.
Key words: finite element method, adaptive method, aluminium extrusion, stress-strain analysis

1. INTRODUCTION

The two main ingredients of an adaptive procedure are:
- a tool for assessing the error of the solution computed with a given mesh and
- an algorithm to define a new spatial discretization.

Two different approaches may be used for assessing the error: error estimators or error indicators. Error estimators approximate a measure of the actual error in a given norm. In this paper, the term error estimator means that the estimated error can be arbitrarily close to the true error. Other definitions are also standard, in some works error estimators are required to behave as equivalent norms of the actual error [16,17,18,33]. Error indicators, on the other hand, are based on heuristic considerations. For each particular application, a readily available quantity is chosen, in an ad-hoc manner as an indicator of error.

Error estimators may be classified into two groups: flux projection estimators and residual type estimators. Most estimators are well defined for linear problems but not for nonlinear problems. For instance, the popular Zienkiewicz and Zhu error estimator for linear problems is only an error indicator for nonlinear problems, because it is based on super convergence properties that cannot be automatically extended to the nonlinear regime. Various choices of an error indicator can be found in the literature. From a geometrical point of view, for instance, the element aspect ratio or, more generally, the distortion can be used [5,14,15]. In nonlinear solid mechanics, some common choices are the equivalent plastic strain or its gradient.

Error indicators are attractive because of their simplicity: they are based on very simple intuitive considerations (geometrical, mechanical etc.) and can be computed easily and efficiently. Quantities used as error indicators are always readily available in the finite element computation, so the overhead cost is minimum. The drawback is that they are heuristic the judgment of the user for defining a proper error indicator for a given problem is critical. Of course, error indicators are very specific of each particular application, and they must be calibrated (with the help of either analytical solutions in simple tests or error estimators). Moreover error indicators only give relative information. Since the error is not quantified, an error indicator only tells where the spatial discretization must be richer, but not how much richer should it be.

Error estimators, on the other hand, must be based on firm mathematical foundations and are usually more expensive to evaluate than error indicators. In exchange for that, they have a major advantage: they provide an objective and quantitative information about the error. Moreover, the range of applicability of a certain error estimator is larger than for a given error indicator [19,32].

The second ingredient of an adaptive procedure is the definition of a new spatial discretization. The goal is to increase or decrease the richness of the interpolation according to the output of the error assessment. Three main types of strategies may be used: h - adaptivity, p - adaptivity and r-adaptivity.

h - adaptivity consists of building a new mesh, using the same type of elements, and adapting the element size to the requirements of the solution. That is, reducing their size where the interpolation must be enriched (i.e. more accuracy is needed) and enlarging the elements where it is already accurate enough.

In p - adaptivity, the degree of the interpolating polynomials is increased. The idea of p - adaptivity is to increase the order of the polynomials where a richer interpolation is needed_ and maintain polynomials of low order where it is already rich enough.

r - adaptivity consists on relocating the nodes to adapt the mesh to the requirements of the solution. The number of nodes and the mesh connectivity remain constant. Nodes are concentrated in zones where they are most needed. The mesh is allowed to coarse in other parts of the domain, where a poorer interpolation suffices.

2. STRUCTURAL NONLINEAR MODEL, ALE METHOD

Ordinary finite deformation problems in solid mechanics have been solved using a Lagrangian method for the finite element mesh. In this method, the finite element mesh is append in, and moves with the material constituting the continuum. The pure Lagrangian approach has the advantage of having to satisfy less complex governing equations, compared to the pure Eulerian method. This may be explain to the absence of the convection terms, and also simple updating technique for path and history dependent materials in the former approach. However, a significant limitation of this description is encountered when the solid deformation become large. Drawback of control over the mesh movement results in distorted meshes with large changes in element dimensions, which adversely affects the accuracy of the solution. Moreover, problem involving certain contact boundaries, especially those with sharp edges or corners, may not be represented precisely in this description. This is due to the fact that the boundary condition has to be specified

on a material point which might move itself.
Despite the introduction of sophisticated remeshing schemes to circumvent the problems associated with excessive mesh distortion in the Lagrangian description, an accurate contact boundary conditions is often questionable. Also, in most cases, remeshing after every time step becomes very expensive.
The Eulerian method in which the mesh remains stationary, on the other hand, introduces other difficulties like appropriate representation of the free boundary. In spite of its capability to represent internal deformation effectively, the Eulerian mesh is less suited for domains whose boundaries or interfaces move. The shortcomings of each of the above method encountered in large deformation analysis, demand a method that can combine the advantages of both the above method into a single method. Such method in which the finite element mesh need not adhere to the material but may be in general motion relative to the material is known as the arbitrary Lagrangian Eulerian method (ALE). The ALE method reserves the potential to represent a Lagrangian or Eulerian method as limiting cases at points, where such method are desired. Thus, it is evident that an ALE method is ideally suited for solving of complex problems in solid mechanics, particularly those dealing with large deformation metal forming, especially for extrusion aluminium technology for hollow parts [24,25,26].
There are explicit and implicit methods in order to solve this problems. Explicit methods, though noted for their efficient performance with a class of structural problems, especially those with high frequency response such as in the case of impact and shock, suffer from the lack of generality of application. In general, the time steps in explicit methods are restricted by stringent stability conditions, which require the evolution of the maximum eigenvalue of the system. Frequently, this is rather inconvenient in nonlinear problems, as the eingevalues change with the evolution of solution. Belytschko has obtained bounds on the eigenvalues based on an evaluation of Raleigh quotient and has stated that very often these upper bounds are drastically overestimated leading to much smaller time steps than are required for stability. Consequently, implicit methods, which have more flexibility with respect to numerical stability, are often more suited in the finite element analysis of inertial problems [22,23,28,29].
A major drawbeck of the implicite method is the evaluation of the tangent iteration matrix during each iteration step performed for achieving dynamic equilibrium. However, the superlinearly convergent quasi-Newton methods have proven to be extremely beneficial in alleviating these obstacles associated with implicit algorithms. These methods replace the Jacobian matrix in Newton's method with an approximation matrix that is updated by mere matrix multiplication at each iteration step resulting in a drastic reduction in the computing effort.
From the previous discussion on time integration, it is apparent, that in any numerical scheme employed for the analysis of elasto-viscoplastic problems, integration of the rate constitutive equations to obtain the internal variables and stresses is of utmost importance. In particular, objective rate forms of stresses and internal variables have to be integrated to yield their updated values at the end of a time step in an increment formulation.

2.1. Governing equations in the ALE method

The ALE method introduces a references configuration which consist of a set of grid points in arbitrary motion in space. Each point of this reference configuration may be unambiguously identified by an invariant set of three independent coordinates χ_i . The motion of the reference frame may then be expressed as an arbitrary continuous function of χ_i and the time t :

$$x_i = x_i(\chi_j, t) \tag{1}$$

The formulation requires an inverse to (1) to exist,

$$\chi_i = \chi_i(x_j, t) \tag{2}$$

such that the Jacobian $J = \left|\frac{\partial x_i}{\partial \chi_j}\right|$ is non vanishing. The above description of the referential system does not refer directly to the motion of particles constituting the continuum and it is therefore necessary to establish a correlation between this system and the Lagrangian system which inherently defines the motion of the particles. Such a relation demands a unique connection between the two systems at every stage of deformation of the continuum, i.e., a single material point may coincide with only one point (not necessarily the same point always) of the grid system. In the Lagrangina system the material particles are identified by a set of coordinates X known as the material coordinates. In the deformed configuration the material particle may occupy a position $x = (x_1, x_2, x_3)$, which may be expressed in terms of the material coordinates and time t as :

$$x_i = x_i(X_j, t) \tag{3}$$

where the inverse of (3) is assumed to exist such that the determinant of the Jacobian $\hat{J} = \left|\frac{\partial x_i}{\partial X_j}\right|$ is not equal to zero. To obtain a relation between the referential and material coordinates, we assume a point x in space occupied at a time t by elements of booth the systems (material point and grid point). Thus, by virtue of the existence of the inverse mapping of (1) and (3), an interrelation between the motion of material and referential (grid) points may be established as:

$$\chi_i = f_i(x_j, t) = \hat{f}_i(X_k, t) \tag{4}$$

Equation (4) may be interpreted as a mapping of the material domain onto the referential domain. With the notion that such a relation exists, it is then possible to derive the conservation equations of mechanics with respect to a referential configuration. The Rejnolds transport theorem is applied to an arbitrary control volume moving through a deforming continuum which is then shrunk to a point. This yields the resulting field equations in the arbitrary Lagrangian – Eulerian method as [5,6,9,25]:

Continuity :

$$\left.\frac{\partial(\rho J)}{\partial t}\right|_{\chi} + J\frac{\partial}{\partial x_i}\left(\rho\left(V_i - W_i\right)\right) = 0 \tag{5}$$

Momentum :

$$\rho G_i + \frac{\partial \sigma_{ji}}{\partial x_j} = \rho\left.\frac{\partial V_i}{\partial t}\right|_{\chi} + \rho\left(V_i - W_j\right)\frac{\partial V_i}{\partial x_j} \tag{6}$$

Energy :

$$\rho\left.\frac{\partial \hat{u}}{\partial t}\right|_{\chi} + \rho\left(V_i - W_j\right)\frac{\partial \hat{u}}{\partial x_j} = \sigma_{ji}D_{ji} - \frac{\partial q_i}{\partial x_i} + f \tag{7}$$

where : ρ - material density, σ_{ij} – Cauchy stress tensor, G – body force vector per units mass, $\hat{u}$ - specific internal energy, q – heat flow vector, D – rate of deformation tensor, f – rate of heat addition by a source, $W\left(=\left.\frac{\partial x}{\partial t}\right|_{\chi}\right)$ - velocity of a representative grid point, $V\left(=\left.\frac{\partial x}{\partial t}\right|_{x}\right)$ - represents the velocity of material point coinciding with the grid point at time t.

The above set of equations may be interpreted as material conservation laws with respect to arbitrarily moving grid points. Advantage of the ALE method is that, in general, the velocity W of the reference frame (representative grid point) may not be the same as the coincident material point velocity V as in the Lagrangian description, or be zero as in the Eulerian method. In the event, that a grid point may coincide with material point, the relative velocity term (W - V) become zero, resulting in the vanishing of the convection terms and consequently the set of equations becomes Lagrangian. Similarly, a pure Eulerian description is obtained by simple setting W = 0.

It is this flexibility to use a different method pointwise, where the grid points may be made to move with material or remain stationary in space or even to move with any arbitrary velocity, that makes the ALE method so attractive. This feature provides a greater scope for the user to manage the nodal or grid points effectively in a large deformation analysis. [3,4,24,25].

When analyzing problems in solid mechanics, especially nonlinear materials with time and history dependent variables using the ALE method, it can be observed from (5) – (7) that a relationship between time derivatives, with material and referential coordinates fixed, is necessary. This relation may be obtained by direct comparison of the ALE conservations laws with the corresponding Lagrangian form (W = V). If β is a time-dependent variable for the material points, then the following relation may be established :

$$\left.\frac{\partial \beta}{\partial t}\right|_{\chi} = \left.\frac{\partial \beta}{\partial t}\right|_{\chi} + \left(W_k - V_k\right)\frac{\partial \beta}{\partial x_k} \tag{8}$$

This equation provides the link necessary for updating various material variables like velocity, density, temperature, Cauchy's stress, etc., to the grid points at the end of each time step in ALE method. The above set equations, together with the constitutive relations discussed in the next section form a basis for the weak forms used in the finite element formulation for large deformation analysis using the arbitrary Lagrangian – Eulerian description.

2.2. Constitutive equations

The constitutive relations for elastic – viscoplastic materials adopted in this paper a rate type, based on the additive decomposition of the rate of deformation into elastic and inelastic parts. This requires the choice of an appropriate objective stress rate, which depends on the class of materials being considered. In this analysis, a general formulation has been carried out for elastically transversely isotropic material. Constitutive relation together with a yield criterion is best represented in a rotated Lagrangian system. In this method, the stress rate is derived from the material time derivative of a stress tensor (also noted as the rotated Cauchy stress tensor) transformed into a rotated space of the material deformation to yield:

$$t = R^T \sigma R \tag{9}$$

where : t - is the rotated stress tensor, σ - is the Cauchy stress tensor, R – is the proper orthogonal tensor obtained from polar decomposition of the deformation gradient tensor R ($R = FU^{-1}$). The rotated stress tensor t, which has the same principal invariants as the Cauchy stress tensor, is the true stress which would result if the deformation gradient was the stretch U alone. An interesting consequence of this transformation is that the conjugate strain rate for this stress is the rotated rate of deformation tensor d, ($d = R^T DR$, where D is the rate of deformation tensor) which, when integrated in time, yields a physically meaningful strain measure [1,2,20,21,24,25]. The frame invariant constitutive model is therefore formulated in a rotated stress-strain space.

In this model, it is assumed that the rotated rate of deformation tensor d admits an additive decomposition into elastic and inelastic parts. A thermodinamic argument for such decomposition is provided within the framework of internal variable theory, where such a decomposition may be obtained independent by any kinematic considerations. Thus:

$$d = d^e + d^{vp} \tag{10}$$

where d^e - represent the elastic part and d^{vp} - represent the viscoplastic part of the deformation rate tensor. The material time derivative of rotated Cauchy stress may then be related to the rate of rotated deformation tensor through the elasticity relation as :

$$\dot{t} = E : \left(d - d^{vp}\right) \tag{11}$$

where E is a fourth order elasticity tensor which may be a function of the spatial stress tensor in general. The above equations not valid for materials which exhibit hyperelastic response but is typical of hypoelastic materials which do not admit a stored energy [44,45].

In the characterization of the viscoplastic part of the rate of deformation, an internal state variable approach is adopted, where the response functions depend on the past history of the independent variables only through the present values of certain variables describing the internal state of the continuum. Within this approach, there exists two categories of models: theories that assume yield criteria which separate purely elastic deformations from combined elastic-viscoplastic deformations and theories that assume no yield criterion and allow the possible existence of elastic and viscoplastic deformations at all stages of loading. It should be noted that the power law model has been widely used by material scientists to simulate metals [10].

However, a recent critical study indicates that the yield based theories in general offer a greater promise in modeling a wide variety of inelastic behavior like loading / unloading, rate and history effects, isotropic and kinematic hardening with Bauchinger effects, creep / relaxation, etc. A three dimensional unified, overstress model of this category [5,6,9,12] has been appropriately extended to include the effects of large deformation and initial anisotropy. In this description, the rate of rotated viscoplastic deformation tensor is given as :

$$d_{ij}^{vp} = \gamma\left\langle \Phi\left(\bar{t} - \kappa\left(W_p, \theta\right)\right)\right\rangle \partial\bar{t}/\partial t_{ij} \tag{12}$$

where γ is a temperature dependent viscosity coefficient, $\langle\ \rangle$ is the MacCaulay operator, κ is an internal state variable, in particular a parameter describing isotropic work hardening, W_p is the inelastic work and θ is the absolute temperature. The term $\bar{t}$ may be defined as an effective rotated Cauchy stress which may be written as :

$$\bar{t} = \sqrt{\tfrac{1}{2}\begin{pmatrix}(F+G)t_{11}^2 + (F+G)t_{22}^2 + 2Gt_{33}^2 - 2Ft_{11}t_{22} - 2Gt_{22}t_{33} - 2Gt_{11}t_{33} + \\ + 2(G+2F)t_{12}^2 + 6Mt_{23}^2 + 6Mt_{31}^2\end{pmatrix}} \tag{13}$$

for transversely isotropic materials. The scalars F, G and M are parameters of anisotropy which depend on the amount of inelastic strain accumulation and the temperature [12,42,43,44].

The rate of deformation tensor defined by (12) is for associated plastic flow and therefore satisfies the normality and incompressibility conditions. The viscoplastic model, though incorporates isotropic hardening, does not include kinematic hardening.

The set of constitutive relations (12) will be complete by supplementing them with the evolution laws for the internal variables. The rate-dependent characterization of plasticity will reside in the characterization of these evolution laws. The growth law used in this model assumes a linear relation between the rate of change of the work hardening parameter κ and equivalent rate of viscoplastic deformation tensor :

$$\frac{\partial \kappa}{\partial t} = E_p \bar{d}^{vp} \tag{14}$$

where E_p is an equivalent plastic modulus. Despite the fact that other nonlinear forms may be more realistic from a representation point of view, the evaluation of the anisotropic parameters are simplified with the linearity assumption. Thus, the set of equations 11-14 form the basis of the material model used in this analysis.

2.3. Weak forms and finite element approximation in the ALE method

Weak forms of the mass and momentum equations may be obtained by taking the product of (5) and (6), respectively, with appropriate weighting functions and integrating over the current grid volume. However, in the finite element analysis of time-dependent finite deformation problems, emphasis is placed on the proper time integration scheme and hence the selection of the appropriate configuration. It is apparent from the discussion given in the introduction that an implicit time integration scheme is better suited for the class of internal problems considered here. In this analysis, we have chosen to neglect the transient term $\rho\partial V_i/\partial t|_\chi$ in the momentum balance equation (6), i.e. we have assumed a quasi-static process with respect to a grid point [42,43].

An overall stability analysis for implicit schemes used in the nonlinear initial value problem with an ALE description is extremely difficult because of the nonlinear, nonsymmetric convection operators together with the nonlinearities arising out of the material constitutive relations. The selection of the implicit method has therefore been based on critical examination of the conclusions drawn from the stability analysis of a few associated systems [44,45].

In problems of fluid mechanics, the convection term in an Eulerian method gives rise to an oscillatory solution when the Galerkin formulation is used. In the recent years various upwinding technique like "upstream weighting", "Petrov - Galerkin finite element method" have been used to stabilize the solution. It is observed that the oscillatory behaviour is apparent when the local Peclet number which is proportional to the velocity becomes large ≈ 2. In solid mechanics problems, such stabilizing may therefore be necessary when treating highly transient problems like impact, shock, etc. However, in the class of internal problems that are of primary interest here, the difference between the grid and the material velocity is not believed to be large enough to initiate significant oscillations in the solution. Nevertheless, it is also reassuring to notice that an energy analysis of the algorithms mentioned above for certain convective systems with nonlinear terms guarantees unconditional stability for the generalized mid-point family [24,25,42].

Using the generalized midpoint rule, the principle of virtual work is written in an intermediate grid configuration $\Omega(t_{n+\alpha})$ as:

$$\int_{\Omega(t_{n+\alpha})} \sigma_{ji} \frac{\partial \bar{u}_i}{\partial x_j} d\Omega + \int_{\Omega(t_{n+\alpha})} \rho (V_i - W_j) \frac{\partial V_i}{\partial x_j} \bar{u}_i d\Omega = \int_{\Omega(t_{n+\alpha})} \rho G_i \bar{u}_i d\Omega + \int_{\Gamma(t_{n+\alpha})} T_i \bar{u}_i d\Gamma \tag{15}$$

where all variables are at the mid-step configuration, corresponding to time $t_{n+\alpha}$ with the position of grid being interpolated as :

$$x^{n+\alpha} = (1-\alpha)x^n + \alpha x^{n+1} \quad \text{for} \quad 0 \le \alpha \le 1 \tag{16}$$

As mentioned, an unambigous connection is required between the lagrangian and the referential system. This implies that the boundary of the grid domain never leaves the boundary of the solid or quantitatively:

$$(V - W) \cdot n = 0 \tag{17}$$

where n is the outward normal to the grid boundary. This enables the accomodationof all kinds of boundary conditions. The boundary of the grid is assumed to be the union of displacement, traction and contact types, i.e. $\Gamma(t) = \Gamma_{iU}(t) \cup \Gamma_{iT}(t) \cup \Gamma_{iC}$ for $1 \le i \le 3$, with the usual respective conditions imposed. In special cases, such as boundaries with concetrated loads it might be necessary to make the corresponding load application points Lagrangian.

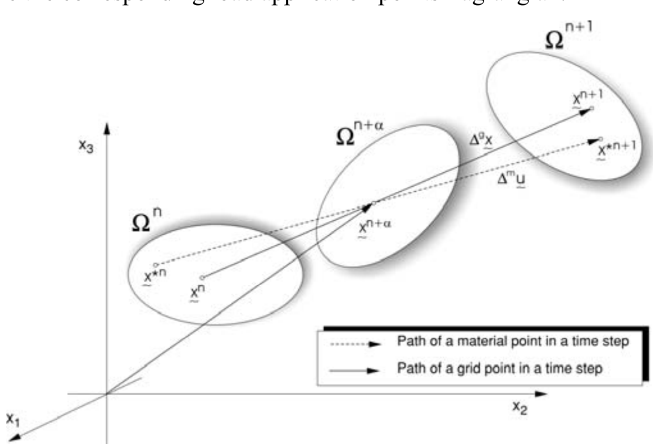


Fig 1. Motion of grid point and material point in a time step in the ALE description

Obviously, the above weak form (15) demands the evaluation of the different variables in the intermediate equilibrium configuration occurring between two consecutive equilibrium states. Let β^n and β^{n+1} correspond to values of any such variables of the problem at time t_n and t_{n+1} , respectively. The midstep value of this variable may then be obtained by a linear interpolation between its two end values as :

$$\beta^{n+\alpha} = (1-\alpha)\beta^{n} + \alpha\beta^{n+1} \tag{18}$$

It is assumed that the value of the variable is known at a grid point in the n-th configuration. Its value in the (n + 1) – th grid configuration may be calculated through the evaluation of the grid point increment of the variable $\Delta^{g}\beta$, by using the generalized midpoint integration as :

$$\Delta^{g}\beta = \beta^{n+1} - \beta^{n} = \Delta t\left(\left.\frac{\partial\beta}{\partial t}\right|_{\chi}\right)^{n+\alpha} \tag{19}$$

However, for path-dependent material variables like Cauchy stress, velocity, work hardening parameter, etc., the evaluation procedure of the increment $\Delta^{g}\beta$ is not direct. For this type of variables, the grid point increment must be derived from the material point increment $\Delta^{m}\beta$. The relation between the time derivatives of a variable at a fixed material point and a grid point (8) may be numerically integrated to yield this desired relation as:

$$\Delta^{g}\beta = \Delta^{m}\beta + \Delta t\left(W_{k}^{n+\alpha} - V_{k}^{n+\alpha}\right)\frac{\partial\beta^{n+\alpha}}{\partial x_{k}^{n+\alpha}} \tag{20}$$

Figure 1 clearly shows the distinction between the two increments. The contact boundary is incorporated using the well-known exterior penalty formulation for unilateral contact problems. It should be emphasized that, though in the present work only normal contact with a rigid surface is trivial. An extensive treatment of contact of two deformable surfaces is trivial.

The central idea of this formulation lies in the introduction of interface spring to preclude penetration. The possible contact boundary Γ_C in this formulation is assumed to consist of purely Lagrangian nodes. Constructing a local coordinate system θ_ξ ($1 \le \xi \le 3$) on each part of the candidate contact boundary, a procedure similar to that discussed [23,24,42,43] may be adopted to reach the following non-penetration condition at the intermediate configuration:

$$\psi^{n+\alpha}\left(\theta_{1}^{n} + \alpha\Delta u_{1}, \theta_{2}^{n} + \alpha\Delta u_{2}\right) \ge \theta_{3}^{n} + \alpha\Delta u_{3} \tag{21}$$

In (21) $\psi^{n+\alpha}(\theta_1 , \theta_2)$ is the parametric representation of the rigid surface at time $t_{n+\alpha}$ and θ_ξ^{n} represents the coordinates of a point on the contact boundary at time t_n . It is assumed that the positive θ_3 coordinate is roughly in the direction of the inward normal to the rigid surface. Linearazition of the function ψ in $\alpha\Delta u_1$ and $\alpha\Delta u_2$ results in the following contact condition:

$$\alpha\Delta u \cdot n^{n+\alpha} - g \le 0 \tag{22}$$

where $n^{n+\alpha}$ is the inward normal to the rigid surface at the intermediate step and g is the gap function. The constraint condition may than be incorporated in the principle of virtual work (15) through the use of an exterior penalty method to yield the approximate approximation for traction as:

$$T^{n+\alpha} = T^{n} + \alpha\Delta T_{n} n^{n+\alpha} = T^{n} + k_{n}\langle\alpha\Delta u_{n} - g\rangle n^{n+\alpha} \quad \text{on} \quad \Gamma_C \tag{23}$$

where $\langle\ \rangle$ is the MacCaulay operator. The above equation (23) is a result of the assumption that the contribution to traction within a time step, is due to normal contact alone. The penalty parameter k_n ($\to\infty$) is representative of the rigidity of stiff normal springs on the contact boundary. It is evident from (23) that only in the case of contact an increment of pressure is activated [34].

The midstep value of the density is evaluated directly by solving the weak form of the continuity equation. This equation, after the application of the constraint condition (17) may be written as

$$\int_{\Omega(t_{n+\alpha})} \left.\frac{\partial\rho}{\partial t}\right|_{\chi} \overline{\rho}\, d\Omega = \int_{\Omega(t_{n+\alpha})} \rho\left(V_j - W_j\right)\frac{\partial\overline{\rho}}{\partial x_j} d\Omega - \int_{\Omega(t_{n+\alpha})} \rho\frac{\partial W_i}{\partial x_i}\overline{\rho}\, d\Omega \qquad (24)$$

Equation (24) is solved for $\left(\partial\rho/\partial t|_{\chi}\right)^{n+\alpha}$ from which density at the intermediate step may be obtained as:

$$\rho^{n+\alpha} = \rho^{n} + \alpha\Delta t\left(\frac{\partial\rho}{\partial t}\right)^{n+\alpha} \qquad (25)$$

The solution to the temperature problem may similarly be obtained easily by considering the weak form of the energy balance equation (7), but it has not been considered in the present work. The set of equations discussed above, must now be solved after finite element discretization using iterative technique.

3. NONLINEAR MODEL OF METAL FLOW FOR ALUMINIUM FORWARD EXTRUSION OF HOLLOW ELEMENTS

Modelling of metal flow process and fulfill of tool for aluminium and their alloys in the area of tool die represent distinct nonlinear proposition. There are meaning changes of cross section from workpiece to finished part which follow high deformation grade. The process of forward extrusion for elements with complete cross section does not matter this investigation. Again, process of forward extrusion for hollow and half hollow elements of aluminium, that is more complex, is object of this investigation. The tool model for this kind of finished part suppose presence central thorn i.e. mandrel, which provide to obtain hollow element with define geometry and shape on outlet part of tool [30,31].

In many case, there are cylindrical workpiece with complete cross section which transform in complex cylindrical and prismatic profile with more and small deviation from basic shape. In the outlet zone of tool, it can be suppose plane deformation state. Such model to impose as solution because the outlet profile has very small thickness, about 1mm, and notable main width.

In this analysis used nonlinear finite element with name Element 11 [MSC.Marc], arbitrary quadrilateral isoperimetric element, with four node, which recommend for plane deformation state. The nodes numbering must be counterclockwise.

Their full geometry is defined with twelve nodes. If some of nodes on side of element is non define, the side retain first shape or rectilinear shape, without of node. Quadrilateral element it was generate by copy single quadrant with polynomial of third rang.

For this reason, it was use for curved boundary surface. As this element uses bilinear interpolation functions, the strains tend to be constant throughout the element. This results in a poor representation of shear behavior. The shear (or bending) characteristics can be improved by using alternative interpolation functions. That is particularly useful for analysis of approximately incompressible materials and for analysis of structures in the fully plastic range. The stiffness of this element is formed using four - point Gaussian integration. This element can be used for all constitutive relations [39].

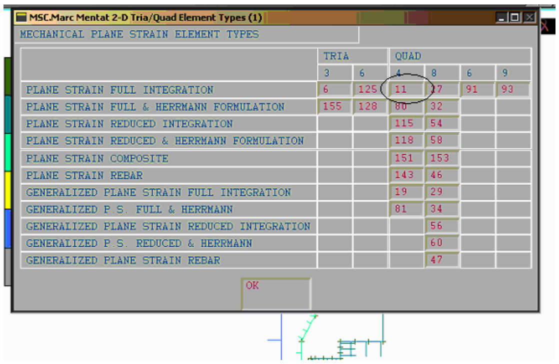
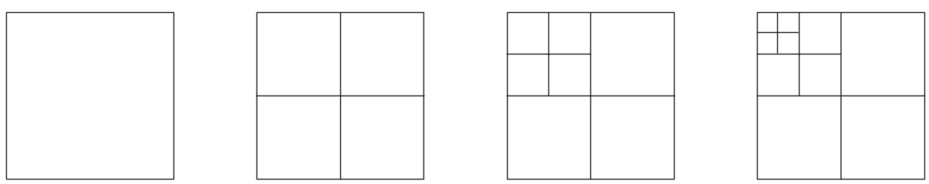


Fig.2. Selection of finite element type

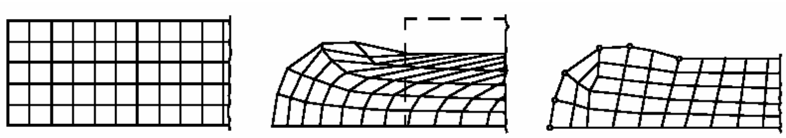
Capability for adaptive behavior of mesh, at this solution, allude augmentation number of elements, same type, by divided original element his inside on define level (fig.3).



Original element, (1) Level 1, (4) Level 2, (16) Level 3, (64)

Fig 3. Adaptive behaviour of element

Anyone adaptive process executed according to full defined and chouse criterion to fulfill in each step. It is possible to choose one, two, three, or four remeshing criteria: Element Distorsion, Contact Penetration, Increment, or Angle Deviation.



no deformed mesh deformed mesh before and after rezoning operation

Fig.4. Rezoning of deformed mesh

There are three sources of nonlinearity: material, geometric, and nonlinear boundary conditions. Material nonlinearity results from the nonlinear relationship between stresses and strains as inherent effect of strain hardening during metal forming process. Geometric nonlinearity results from the nonlinear relationship between strains and displacements on the one hand and the nonlinear relation between stresses and forces on the other hand. Boundary conditions and/or loads can also cause nonlinearity i.e. contact and friction problems.

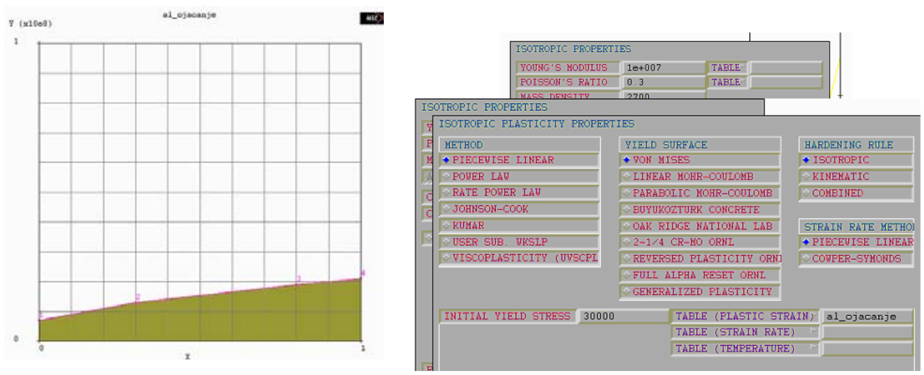


Fig.5. Plasticity parameters of aluminium

Two dimensional model of workpiece with parametrics plasticity during fulfill of tool area and plasticity metal flow in calibration zone which follow, model of rigid matrix with function of boundary of space for metal forming process and model of mandrel which ensure obtain of hollow aluminium elements it can recognize at analysis. The workpiece model has aluminium characteristics well know and strain hardening curve that proposed in flowchart.

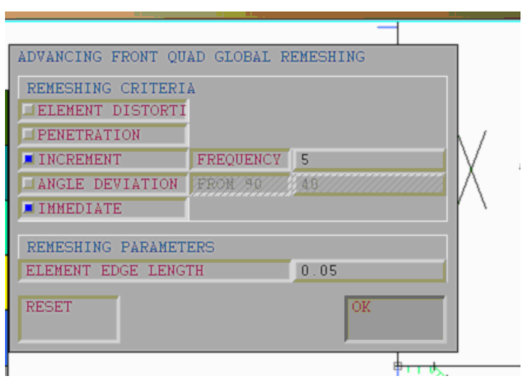


Fig.6. Remeshing parameters

The workpiece relative motion it was task along y-axis with negative sign and enough finite elements (336) for stuff whole tool zone and flow in calibration zone [38,39,40,41]. The quadrilateral element size is change on two level, 0.04 and 0.05 whereby fulfill two opponent demands, refinement of simulation process and their time space. Remeshing criteria of mesh was defined with frequency 5.

Two loading stepping was define in order to get rather solution indicate numerous sample of simulation and analysis, first and very small when metal forming process started and second which take whole process simulation of aluminium plasticity flow. For this reason, first and second loading stepping was establish for aluminium forming process. The most important parameter for attain number of iteration is number of step or time of loading in simulation of metal forming process. In order to get more accurate solution it was make many simulation experiment.

The plane deformation state condition and criterion was defined after loading stteping, their number and time consuming. The Lagrangian update method was selection with maximal number of nodes in contact to 2000. The output results it can be selection according analysis demands.

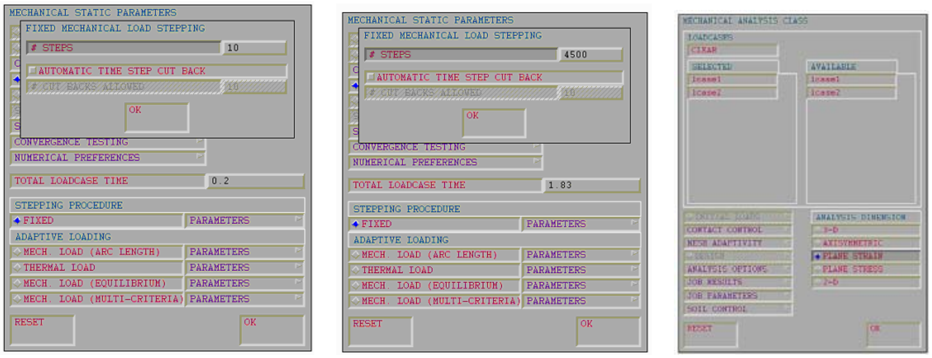


Fig.7. Loading parameters

4. NONLINEAR ADAPTIVE SOLUTIONS FOR FORWARD EXTRUSION OF HOLLOW ELEMNTS

On the next figures shows some results of investigation stress – strain field in the area of tool cavity by nonlinear finite element method [38,39,40,41]. Process simulation of filling tool with aluminium alloy (AlMgSi0.5) at the variable temperature extrusion, with temperature of workpiece on 480°C and temperature of tool die on 450°C, at the constant velocity extrusion 7m/min, shows only characteristic steps and attain level of stress and strain. The mesh behaviour in calibration zone and method adaptivity, remeshing and refinement evident demonstrate indispensable this approach in following of metal forming process.

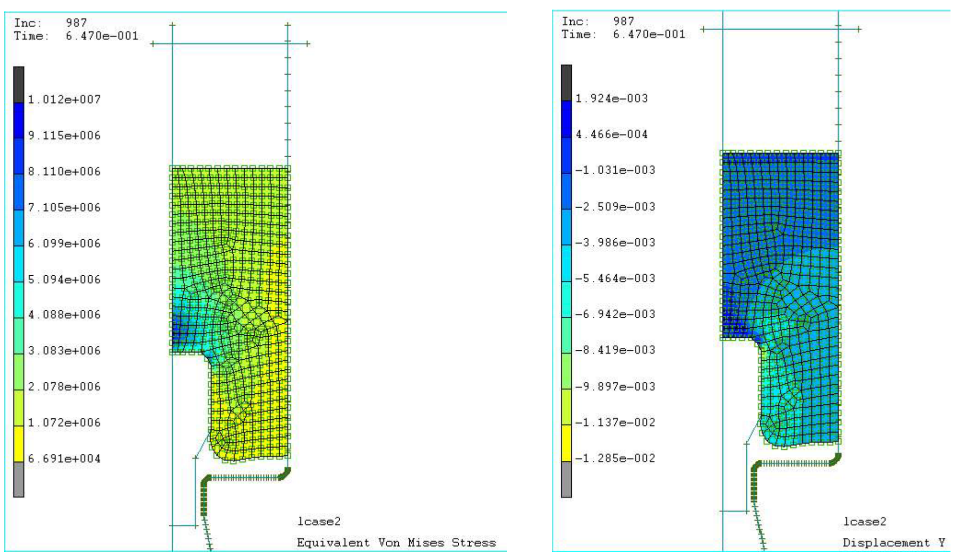


Fig. 8. FEM adaptive model for stress-strain field in tool cavity near mandrel, 987 step

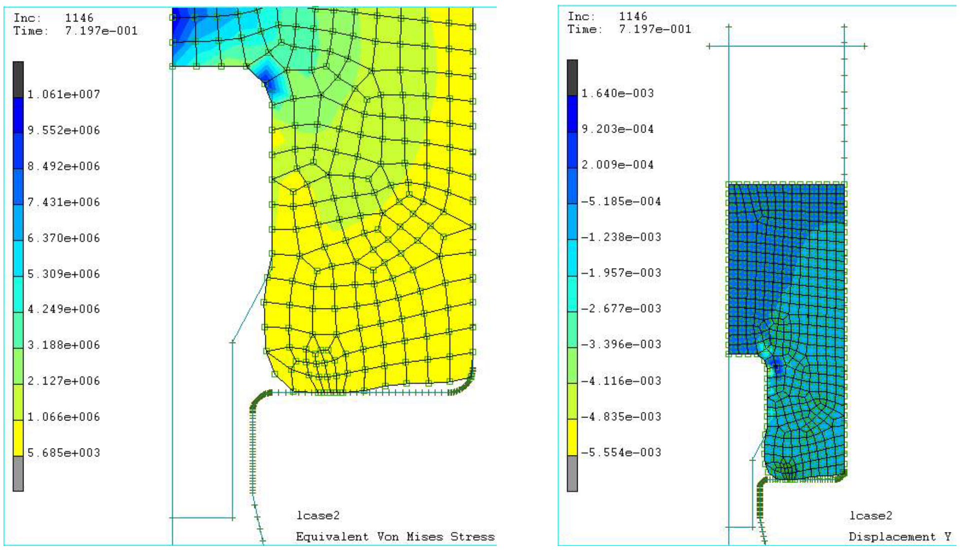


Fig.9. FEM adaptive model for stress-strain field in tool cavity near mandrel, 1146 step

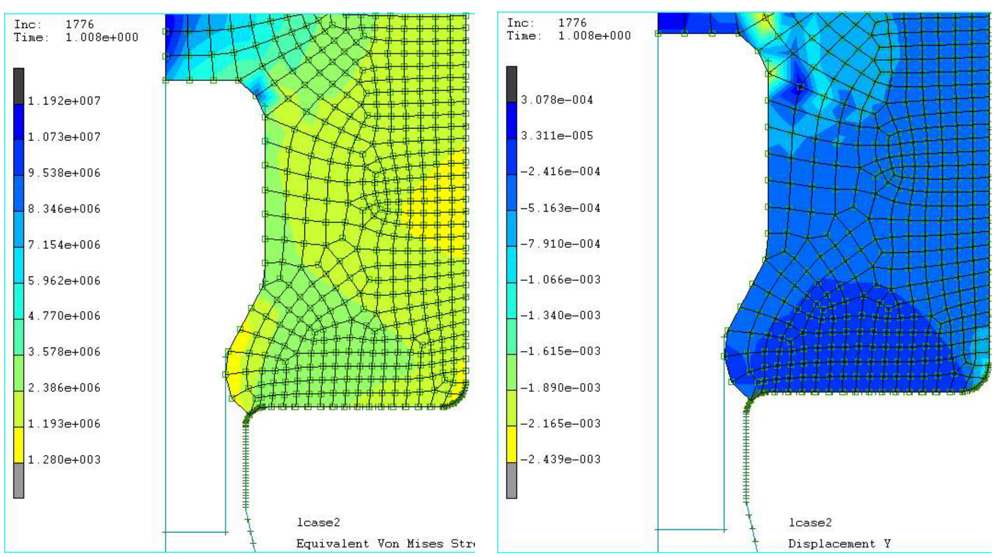


Fig.10. FEM adaptive model for stress-strain field in tool cavity near mandrel, 1776 step

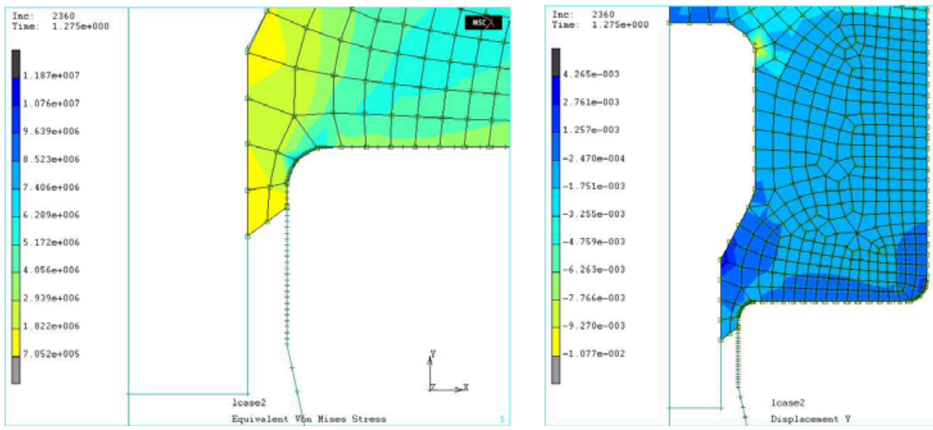


Fig.11. FEM adaptive model for stress-strain field in tool cavity and outlet zone, 2360 step

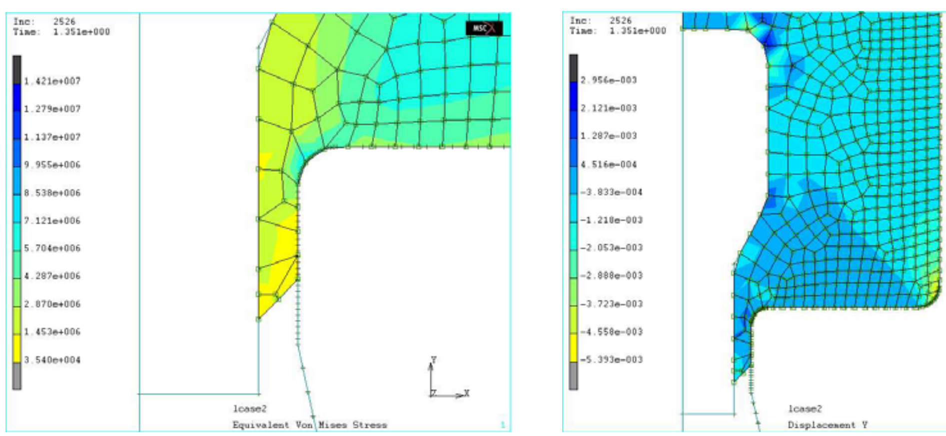


Fig.12. FEM adaptive model for stress-strain field in tool cavity and outlet zone, 2526 step

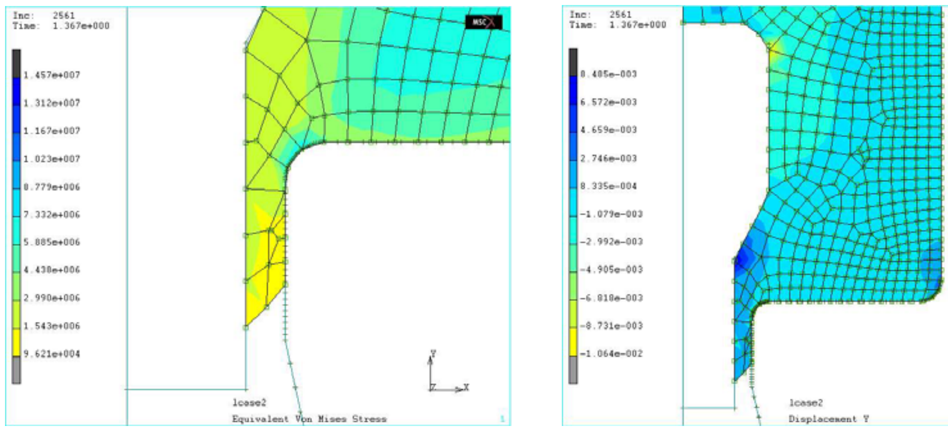


Fig.13. FEM adaptive model for stress-strain field in tool cavity and outlet zone, 2561 step

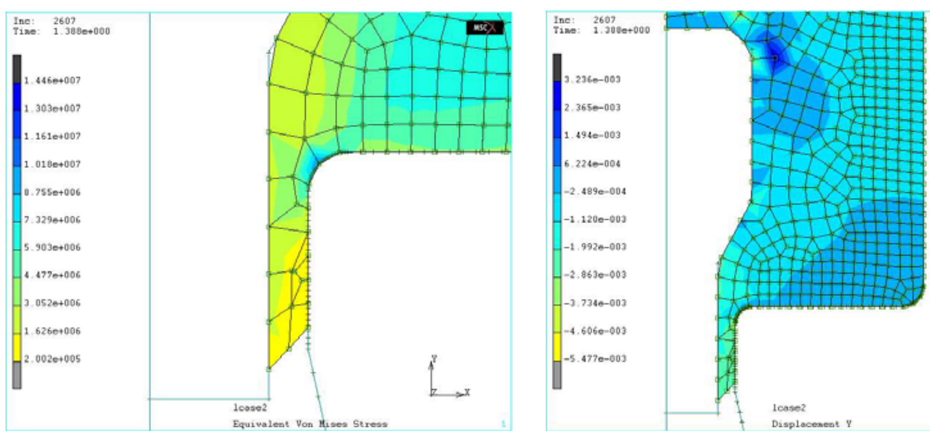


Fig.14. FEM adaptive model for stress-strain field in tool cavity and outlet zone, 2607 step

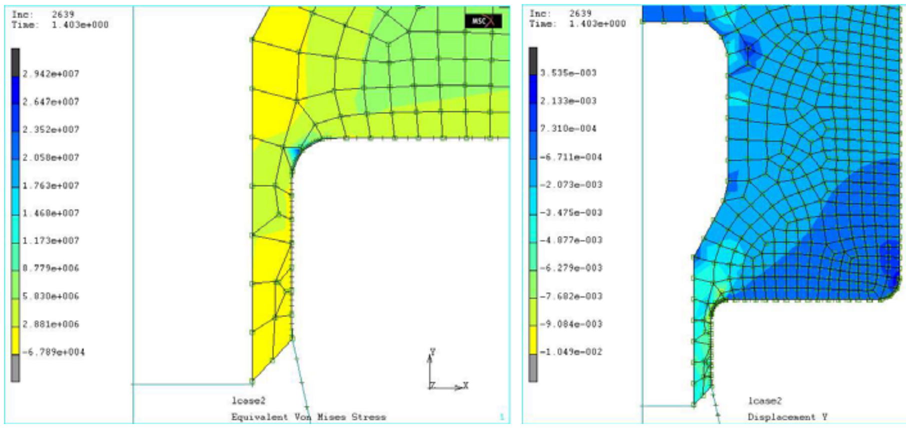


Fig.15. FEM adaptive model for stress-strain field at outlet zone, 2639 step

5. CONCLUSION

The basic postulates of adaptive method indicate the great potential of software tools in a modern technology process. Observing the changes of stress – strain field in tool cavity in this manner, becomes more accessible to analysis of parameters in metal forming processes. There is a potential of metal flow modeling and optimization in die area on a virtual model with stress plasticity control, displacement in x, y directions, friction force etc. Implementation of adaptive model in real conditions of plastic deformation during aluminium forward extrusion of hollow element at high temperatures is presented.

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NELINEARNE ADAPTIVNE METODE KONAČNIH ELEMENATA ZA MODELIRANJE PROCESA ISTOSMERNOG ISTISKIVANJA ŠUPLJIH ELEMENATA

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REZIME

Standardne metode konačnih elemenata danas se zasnivaju na primeni adaptivnih metoda. Za bilo koji problem adaptivnost je osnovni alat za dobijanje numeričkog rešenja sa kontrolisanom tačnošću. Ovaj metod nudi rešenje za praćenje poremećaja u strukturi materijala sa mogućnošću vođenja i kontrole procesa kao i naponsko deformacione analize. Za neke probleme u nelinearnoj oblasti adaptivnost je i više nego fundamentalna metoda bez koje sama metoda konačnih elemenata gubi smisao. To je slučaj i u mehanici čvrstih tela sa posebnom primenom na tehnologiju istiskivanja aluminijuma u samom žarištu deformacije.

Ključne reči: metoda konačnih elemenata, adaptivne metode, istiskivanje aluminijuma, naponsko deformaciona analiza.