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DETRMINING THE TRANSITONAL AREA OF SQUARE CUPS IN OIL HYDRAULIC FORMING PROCESS

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ABSTRACT

The precise stress distribution in the focus of deformation, i.e. in the square cup flanges (of non-axis- symmetric parts), during the process of deep drawing is quite difficult to determine, owing to the fact that during the process of their forming three characteristic areas appear on the flange: corner area, side zone and the transitonal area between them. Different stress-strain states appear in the above mentioned areas. There are six possible combinations of the stress- strain state. An attempt to determine the maximum value of normal stress in radial direction, i.e. the drawing force, by means of a single equation has not given good results.

This paper provides both a theoretical analyses of the stress-strain state on a square cup and the derivation of the expressions leading to the determining of a transitional area value and also resulting in six different equations enabling calculatng of the force forming the square cups in oil-hydraulic process. Practical experimental results have confirmed the validity of the expression enabling determiningthe value of transitional area.

1. INTRODUCTION

Taking into account the stress-strain state occuring in the sqaure cup flange during the formation process, there are three characteristic areas. Deep drawing is performed in all four corners of the square cup. Double angle plate bending occurs in the middle of the side walls (U-profile). To achieve harmonizing of stress-strain states coming from the angles and the middles of side zones there has to be a transitional area between them.

Theoretically, and taking into account the lenght b, the width a of the square cup and the corner radius r_c, as well as the transitional area magnitude g, it is possible to expect six different cases, [1]:

1. $b_{ps} = b - 2r_c - 2g = b_s - 2g > 0$, $a_{ps} = a - 2r_c - 2g = a_s - 2g > 0$ – there is a purely side zone both on the longer and shorter sides of the cup between the two transitional areas, of b_{ps} length, i.e. a_{ps} (Fig. 1, a and Fig. 4, b);
2. $b_{ps} > 0$, $a_{ps} = 0$ – on the longer side of the cup, there are two transitional areas and a purely side zone, whereas on a shorter side the transitional areas merege into each other (there is no purely side zone);
3. $b_{ps} > 0$, $a_{ps} < 0$ – areas occuring on a longer side are the same as in 1 and 2 above, whereas there is larger or shorter overlaping of transitional areas on shorter sides;
4. $b_{ps} = 0$, $a_{ps} = 0$ – one transitional area merges into the other directly on both the shorter and longer sides;
5. $b_{ps} = 0$, $a_{ps} < 0$ – the same situation as in 4 above occurs on longer side, whereas there is overlaping of transitional areas on the shorter one; and
6. $b_{ps} < 0$, $a_{ps} < 0$ – there is overlaping on both the shorter and longer sides.

The distribution of stress and strain in the square cup flange, as well as the expressions to determine the drawing force (pressure) shall vary in different cases. Owing to this it is important to know the magnitude of the transitional area.

2. STRESS STATE IN THE SQUARE CUP FLANGE

During the process of square cups forming three characteristic areas occur on the flange: corner area, side zone and the transitonal area between them. (Fig. 1, a, [1], [2]).

Precise stress distribution in the focus of deformation in the corner, in the process of nonsymmetric cups forming, is quite difficult to determine. Apart from the nominal tensile stress in radial direction σ_ρ and the normal compressive stress in circumferential direction σ_θ (stress-strain state typical for deep drawing of cylindrical cups) there are some shear stresses $\tau_{\rho\theta} = \tau_{\theta\rho}$. Shear stresses occur due to the curve radius changes on the circumference of a cup. The curve radius varies from $r = r_c$ in the corner of the part to $r \to \infty$ on the the side zone. All the stresses are in the function of coordinates ρ i θ (Figure 1, b).

It is considered that there are no shear stress-strain states along the corner axis of symmerty, that is the stress-strain state present is similar to the one observed in deep drawing process of axisymmetric cups. Owing to this, the origin of polar coordinate system is positioned in the center of the curve (O_1) defined by the radius of the cup corner r_c, whereas the angel measuring (coordinate θ) is taken from the axis of symetry of the cup angle, Figure 1, b.

Besides that, it is not only possible to define precisly the outward contour of the flange because it is variable, but also difficult is to determine the real dependence of shear stress icrement $\tau_{\rho\theta}$ commencing at the axis of symetry of the cup corner to the transitional line of the corner area in the side zone ($\theta = \pi/4$). Assuming that the shear stress varies in accordance with a linear dependance and that it is a function of coordinate θ, it follows:

$$\tau_{\rho\theta} = -a \cdot \frac{K}{\sqrt{3}} \cdot \frac{\theta}{\alpha}, \tag{1}$$

where the sign "-" means that the shear stress occurs along the negative direction of ρ-axis, and the expressions to determine the normal stresses in both radial and circumferential directions obtain the following forms respectively, [2]:

$$\sigma_\rho = \frac{2K}{\sqrt{3}} \cdot \left(\sqrt{1 - \frac{a^2 \cdot \theta^2}{\alpha^2}} - \frac{a}{2\alpha} \right) \cdot \ln\left(\frac{R_{fl}}{\rho} \right) \cdot \tag{2}$$

$$\sigma_\theta = \frac{2K}{\sqrt{3}} \cdot \sqrt{1 - \frac{a^2 \cdot \theta^2}{\alpha^2}} - \sigma_\rho \cdot \tag{3}$$

Where: K – specific flow stress,
R_{fl} – radius of the outward flange edge,
α – angle that defines the area to which the focus of plastic yield expand and
a – unknown constant.

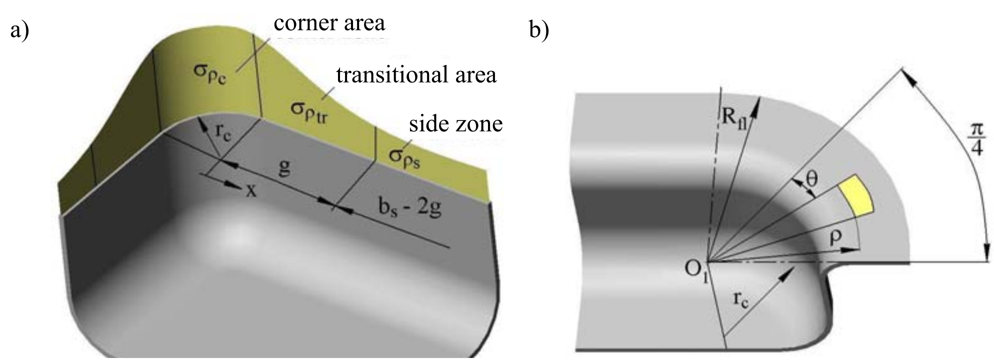


Figure 1: Corner area, side zone and the transitonal area (a) and system of coordinates (b)

If it is assumed that the focus of plastic yield expands only to the corner area of ($\alpha = \pi/4$) and that the maximum possible shear stress $\tau_{\rho\theta} = \tau_{max} = K/\sqrt{3}$ occur along the transitional line of corner zone to side zone of ($\theta = \pi/4$), it results that a = 1 from eguation (1). However, apart from shear yield $\gamma_{\rho\theta}$ occuring along the transitional line of corner zone to the side zone, there are also normal plastic yields in both radijal and circumferential directions φ_ρ i φ_θ, therefore it is obvious that we are not dealing with pure shearing, namely the shear stress can not obtain its maximum value.

To prevent excessive sheet metal drawing in over the side zones of large square cups and the cups with the length/width ratio of b/a > 2.0÷2.5, draw beds are used for industrial purposes.

It is well known that draw beds must not extednd up to the transitional line of corner zone to the side zone ($\theta = \pi/4$, Fig. 1, b). The ends of draw beds are positioned behind the line that makes the angle of 10÷15° (α = 55÷60°) with the transitional line.

Therefore it is adopted that the focus of plasic yield extends to the value of $\alpha \approx 1$ rad $\approx 57.3°$. It is also assumed that the shear stress occuring along the transitional line of corner zone to the side zone obtains a half of its maximum possible value of $\left(\tau_{\rho\theta} = K / 2\sqrt{3}\right)$. Now it is obtained from equation (1) that a = 2/π = 0.6366.

Expressions (1), (2) i (3) are applicable to the the cases of ideal flange plastic forming (the material of sheet metal is isotropic); the whole flange area is engaged in the forming process, whereas the flange width has a constant value; the flange lubrication is ideal, so that there is no die friction, i.e. $\mu \approx 0$; there are no wrinkles on the flange, meaning that no draw bed is needed, therefore plain strain and stress state occur in the flange). The diagrams showing distribution of normal and circumferential stress, obtained according to equations (1), (2) i (3) are presented in Fig. 2.

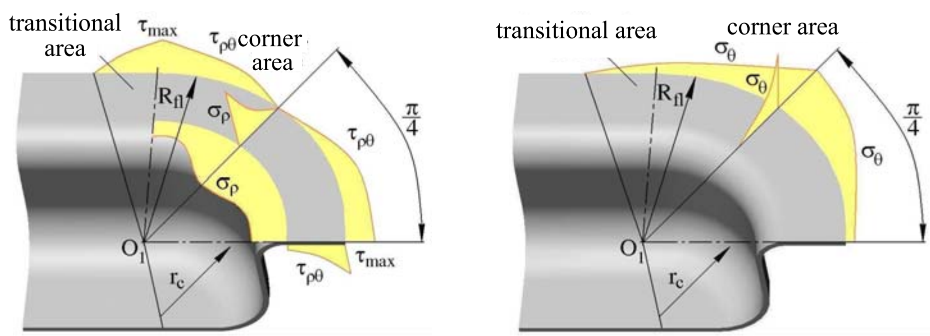


Figure 2: Distribution of normal and shear stresses

Plastic forming within the purely side zones is localized to the area of the rounding radius r_d, where bending is present, so that in the flange area, taken from the theoretical point of view, there are neither any stresses nor plastic forming. It is obvious that there must exist a transitional area between the corner zone and the side zone, where there is a stress reduction coming from the corner zone (Figure 2).

3. THEORETICAL DETERMINING OF TRANSITIONAL AREA MAGNITUDE

The transitional area does not have the same width. It is larger over the outward than the inward flange edge. (Figure 2). The assumption that the flange outward and the inward transitional areas are of the same width will make it possible to determine its magnitude.

An elementary volume within the transitional area is observed (Fig. 3). The strain in circumferential direction and shear yield, resulting from the deformation within the corner area, are neglected so that the elementary volume is affected by the normal stress caused by the pressure in the direction parallel to the inward contour of the die (circumferential direction), the tensile stress in the direction perpendicular to the inward contour (this direction, owing to the analogy with the corner area, will be denoted as a radial direction), then the normal stresses caused by the pressure in the direction perpendicular to the surface of sheet metal and the circumferential contact stress caused by the pressure of the blank holder. The origin of coordinate system O_2 is located in the middle of side zone. The equation denoting balance in circumferential direction is given in the following form [2]:

$$d\sigma_\theta = \frac{2\cdot\tau\cdot dx}{s}\,. \tag{4}$$

Owing to the circumferential contact stress conjugation, it is possible to observe its variation in the direction perpendicular to the inward flange contour. The circumferential contact stress in the rounding die area of radius r_d, taking into account the analogy with the tackle, varies according to the rule given by:

$$\tau = e^{f\cdot\mu\cdot x}\cdot p_{bh}\,. \tag{5}$$

where: f – damping factor, including damping of normal stresses caused by the pressure in the circumferential direction σ_θ, occuring in the corner area;
μ – coefficient of friction in the flange and
p_{bh} – blank holder pressure.

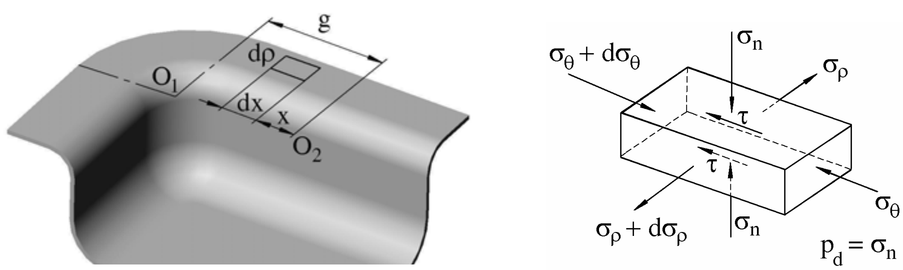


Figure 3: The elementary volume within the transitional area

By substituting the equation (5) in (4), and after its integrating, the following is obtained:

$$\sigma_\theta = \frac{2\cdot p_{bh}\cdot e^{f\cdot\mu\cdot x}}{f\cdot\mu\cdot s} + C\,. \tag{6}$$

The normal stress in radial direction, occuring on the line of corner area transitional to transitional area of (x = g) and the outward flange edge, is equal to zero ($\sigma_\rho = 0$). By applying the the Tresca palstic yield, it is obtained $\sigma_\theta = K$. To be able to take into account the whole flange width, it is assumed that the mean value of the plastic flow stress at the line of corner end is, $\sigma_\theta = K_{mid}$ for the purpose of calculating. When this boundary condition is inserted in equation (6), and having determined the constant of integration, the following is obtained:

$$\sigma_\theta = \frac{2\cdot p_{bh}}{f\cdot\mu\cdot s}\left(e^{f\cdot\mu\cdot x} - e^{f\cdot\mu\cdot g}\right) + K_{mid}\,. \tag{7}$$

The second boundary condition valid for the outward flange edge, is the fact that there are no normal stresses $\sigma_\theta = 0$ caused by pressure on the the line of side zone transition into the transitional area (x=0). By substituting these values in (7), and having performed the logarithmic operations and simple mathematical transformations the following expression is obtained to determine the magnitude of the transitional area:

$$g = \frac{\ln\left(\frac{f \cdot \mu \cdot s}{2 \cdot p_{bh}} \cdot K_{mid} + 1\right)}{f \cdot \mu} . \qquad (8)$$

The minimum blank holder pressure (if the blank holder pressure within the the lower boundary range of a good area is exceeded, apparent wrinkles may occur) to form square cups in an oil-hydraulic process is calculated according to the following expression, [3]:

$$p_{bh} = 0.003 \cdot \left[\left(\frac{D_{0f}}{d_{inf}} - 1\right)^2 + \frac{D_{0f}}{200 \cdot s}\right] \cdot R_m , \qquad (9)$$

where: D_{0f} [mm] - circular blank raduis the surface of which is equal to the sasamples of a part intended for for the drawing of a box cross section,

d_{inf} [mm] - inward diameter of a fictive cylindrical cup, the surface of its cross section is equal to the surface of square cup cross section;

R_m [N/mm^2] - tensile strength of sheet metal.

The mean value of plastic yield K_{mean} is determined in the same way as in the case of cylindrical cups deep drawing (of $d = 2 \cdot r_c$ diameter), which is obtained by putting together the four corner areas into a whole. Consequently, the mean value of plastic yield K_{mean} is determined by the following expression:

$$K_{mean} = \frac{K_0 + K_{max}}{2} , \qquad (10)$$

where: $K_0 = C \cdot 0.002^n$ - plastic yield at the beginning of the plastic forming and

$K_{max} = C \cdot \varphi_{max}^n$ - plastic yield as determined from the flow curve given by the logarithm of plastic forming $\varphi_{max} = \ln \frac{\sqrt{R_0^2 - R_{fl}^2 + r_c^2}}{r_c}$, occuring at the moment of complete grasp of the rounding die edge r_d and the punch r_p.

On the grounds of equation (8) it is possible to conclude that the magnitude of the transitional area is mostly affected by the following: the blank holder pressure p_{bh}, coefficient of friction μ at the cup flange, sheet metal thickness s, as well as cup corner radius r_c and the cup drawn height h, because the mean value of resistance to plastic forming K_{mid} is directly influenced by them.

The only problem appearing in equation (8) is to determine the damping factor f. Experimental results have shown that the value of f = 0.03 taken in standard deep drawing of sqaure cups is sufficient for the smaller punch radii ($r_p = 8 \div 10$ mm) and sheet metal thickness $s_0 < 2.0$ mm, [3].

However, still there is a theoretical possibility to determine the maximum value of transitional area g, in other words to determine the minimum value of damping factor f.

Observation of the wrinkles on the drawn cup flanges led us into this conclusion [4]. Namely, in case of insufficient blank holder pressure, it happened to have wrinkles in the middle of the longer side of the cup drawn (Fig. 4). It means that the magnitude of the transitional area in given cases was at least equal to the half of the side zone lenght $b_s/2$.

Since the length of the cup is larger than its width, i.e. $b \geq a$, in the forming process of square cups of rectangular cross section, then the condition determining the maximum length of the transitional area (in case of complete overlaping of transitional areas) is given by $g_{lim} = a_s = a - 2r_c$, and substituting g_{lim} in (8), the following expression is obtained:

$$f \cdot \mu \cdot g_{\lim} - \ln\left(\frac{f \cdot \mu \cdot s}{2 \cdot p_{bh}} \cdot K_{mean} + 1\right) = 0 \, , \tag{11}$$

The minimum value of a damping factor f is obtained through an iterative procedure applied to this expression.

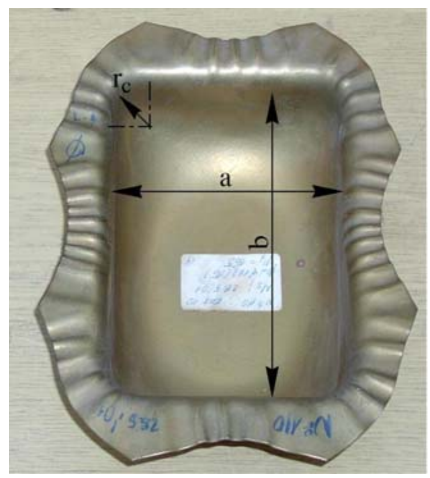
a)  b) 

Figure 4: Square cups of square (a) and rectangular (b) cross sections

The expression (11) was used to determine the value of dampening factor of the square cups formed in three different materials of sheet metal designated as: steel RSt13, ($K = 630 \cdot \varphi^{0.207}$ N/mm^2); brass CuZn37 ($K = 634.1 \cdot \varphi^{0.235}$ N/mm^2) and electrolytic copper E1-Cu58 ($K = 443.1 \cdot \varphi^{0.223}$ N/mm^2). The sheet metal thicknee of s = 1.0 mm was used in each case. The dimension of cups' cross section taken in the calculation were: $a \times b = 80 \times 80$ mm ($c = b/a = 1.0$, sl. 4, a); $a \times b = 80 \times 120$ mm ($c = 1.5$, Fig. 4, b) i $a \times b = 80 \times 160$ mm ($c = 2.0$). The value of $\mu = 0.1$ was adopted for the coefficient of friction. The minimum blank holder pressure p_{bh} (below this value apparent wrinkles form themselves in the flange) was calculated according to (9), whereas the mean value of resistance to plastic forming K_{mean} according to (10). The calculated values of the damping factor are given in Table 1.

Table 1: Values of the damping factor f for maximum length of the transitional area g_{lim}

Corner radius; g_{lim}, [mm]	Cup height, [mm]	Cup length to width ratio c = b/a					
		c = 1.0		c = 1.5		c = 2.0	
		$b_s/2$	f	$b_s/2$	f	$b_s/2$	f
r_c = 12.5 mm; $a_s = g_{lim}$ = 55 mm	h = 30	27.5 mm	0.29÷0.30	47.5 mm	0.29÷0.30	67.5 mm	0.29÷0.30
	h = 45		0.21÷0.22		0.21÷0.22		0.21÷0.22
r_u = 16.0 mm; $a_s = g_{lim}$ = 48 mm	h = 30	24.0 mm	0.36÷0.37	44.0 mm	0.36÷0.37	64.0 mm	0.36÷0.37
	h = 45		0.27÷0.28		0.27÷0.28		0.27÷0.28

It could be seen that the damping factor f does not depend on the cup cross section length to width ratio. Slight differences in damping factor f have been observed for the three materials taken (assuming that other conditions remain the same) For example: f = 0.29 for RSt13 (DIN 17007) and f = 0.30 for E1-Cu58 (DIN 1708).

Since we were dealing with similar cups, the unique value of f = 0.3 for the damping factor was adopted to determine the flange stress-strain state and to calculate the pressure required for square cups drawing in working fluid. The values of transitional area length, as calculated according to expression (8), are given in Table 2.

Table 2: Transitional area length g, [mm] (for f = 0.3) and calculated values of blank holder pressures

Corner radius	Cup height	Material	Cup length to width ratio		
			c = 1.0	c = 1.5	c = 2.0
r_c = 12.5 mm g_{lim} = 55 mm	h = 30 mm	RSt13, DIN 17007	53.92	54.22	53.48
		CuZn37, DIN 17660	53.55	53.85	53.14
		E1-Cu58, DIN 1708	53.71	54.00	53.26
	h = 45 mm	RSt13, DIN 17007	46.75	48.15	48.24
		CuZn37, DIN 17660	46.40	47.80	48.60
		E1-Cu58, DIN 1708	46.55	47.95	48.03
r_c = 16.0 mm g_{lim} = 48 mm	h = 30 mm	RSt13, DIN 17007	53.50 (*)	53.64 (*)	52.90 (*)
		CuZn37, DIN 17660	52.84 (*)	53.16 (*)	52.42 (*)
		E1-Cu58, DIN 1708	53.05 (*)	53.36 (*)	52.62 (*)
	h = 45 mm	RSt13, DIN 17007	46.14	47.57	47.68
		CuZn37, DIN 17660	45.68	47.10	47.20
		E1-Cu58, DIN 1708	45.90	47.30	47.39

It is obvious that for r_c = 16.0 mm and h = 30 mm the calculated values of transitional areas are greater than the values of boundary range ($g > g_{lim}$ = 48 mm). It could be expected, since the values of the daming factor f, shown in Table 1, are the minimum values.

It should also be noted that the magnitudes of transitional areas g were determined for the calculated values of the blank holder preassures (expression (9)), i.e.for the values falling within the range of lower boundary range of good area. Below this boundary values apperent wrinkles form themselves.

Owing to this, the values of blank holder pressure were determined for the cases marked with (*) in Table 2, for the purpose of experimental cups forming. Additional calculation of the transitional area maginitude were performed for the given mean values of blank holder pressures. The results are shown in Table 3, and consequently the length of the transitional area g was smaller than the boundary range value of g_{lim} = 48 mm.

Table 3: Transitional area length g[mm] (for f=0.3 and mean values of blank holder pressure taken in experimental researches [1], [4].

Corner radius, [mm]	Cup height, [mm]	Material	Cup length to width ratio		
			c = 1.0	c = 1.5	c = 2.0
r_c = 16.0	h = 30	RSt13, DIN 17007	(p_{bh}= 1.32 MPa) p_{bh} = 1.5 MPa g = 48.36 mm	(p_{bh}= 1.31 MPa) p_{bh} = 1.8 MPa g = 45.40 mm	(p_{bh}=1.35 MPa) p_{bh} = 2.2 MPa g = 40.55 mm
		CuZn37, DIN 17660	(p_{bh}= 1.28 MPa) p_{bh} = 1.6 MPa g = 46.99 mm	(p_{bh}= 1.26 MPa) p_{bh} = 2.1 MPa g = 43.38 mm	(p_{bh}= 1.30 MPa) p_{bh} =2.0 MPa g = 41.53 mm
		E1-Cu58, DIN 1708	(p_{bh}= 0.91 MPa) p_{bh} = 1.2 MPa g = 45.86 mm	(p_{bh}= 0.90 MPa) p_{bh} = 1.3 MPa g = 43.89 mm	(p_{bh}= 0.92 MPa) p_{bh} = 1.5 MPa g = 40.46 mm

Note: Calculated values of blank holder pressures, according to (9) are given in brackets.

4. CONCLUSION

It is necessary to be familiar with the transitional area magnitude in order to determine the flange stress-strain state of the cups of both square and rectangular cross section, as well as to determine the pressure of the working fluid required for their forming.

It is not possible to make use of one unique expression to determine the pressure of the working fluid (drawing force). The pressure of the working fluid has to be calculated according to six different expressions depending on the transitional area magnitude.

The expression determining the magnitude of the transitional area is derived on the grounds of flange transitional area stress-strain state analyses.

The paper also provides the methodology and the expression enabling calculation of the damping factor required for normal stresses in circumferential direction σ_θ within the transitional area.

Experimental check up of the expression determining the working fluid pressure (required for drawing), including the transitional area magnitude g and the damping factor f = 0.3 has shown a satisfying discrepancy between the calculated and experimental values. The relative error was less than 15%.

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ODREĐIVANJE VELIČINE PRELAZNE OBLASTI PRI OBLIKOVANJU KUTIJASTIH DELOVA NESTIŠLJIVIM FLUIDOM

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REZIME

Tačnu raspodelu napona u žarištu deformacija, tj. na vencu dela, pri dubokom izvlačenju kutijastih (osno-nesimetričnih) delova, veoma je teško odrediti. Ovo zbog toga, što se pri oblikovanju kutijastih delova javljaju tri karakteristične oblasti na vencu dela: oblast ugla, oblast pravolinijske strane i prelazna oblast između njih.

U navedenim oblastima se javljaju različita naponsko-deformaciona stanja. Postoji šest mogućih kombinacija naponsko-deformacionog stanja. Zbog toga i pokušaji da se jednim jedinstvenim izrazom odredi vrednost maksimalnog radijalnog napona, odnosno sile izvlačenja, nisu dali dobre rezultate.

U ovom radu, izvršena je teorijska analiza naponsko-deformacionog stanja na vencu kutijastog dela i izvođenje izraza za određivanje veličine prelazne oblasti, čime je omogućeno dobijanje šest različitih izraza za sračunavanje sile oblikovanja kutijastih delova nestišljivim fluidom.

Takođe, data je i metodologija i izraz za sračunavanje vrednosti faktora prigušenja normalnih napona u tangencijalnom pravcu σ_θ u prelaznoj oblasti.

Pri eksperimentalnoj proveri izraza za pritisak u nestišljivom fluidu (koji je potreban za izvlačenje), a u kojima su uključene vrednosti veličine prelazne oblasti g i faktora prigušenja $f = 0.3$, dobijena je zadovoljavajuća razlika između računskih i eksperimentalnih vrednosti. Relativna greška je bila manja od 15%.