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APPLICATION OF A NON - LOCAL CRITERION TO FRACTURE PREDICTION IN PLANE – STRAIN DRAWING

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ABSTRACT

A popular method for fracture prediction in metal forming processes is based of the concept of workability diagram. However, for some constitutive models direct applications of this method are impossible when fracture occurs at maximum friction surfaces because the velocity field is singular such that the equivalent strain rate approaches infinity at such surfaces. To overcome this difficulty, it is possible to adopt non – local fracture criteria which can also be based on the workability diagram. In the present paper such a criterion is used to predict fracture in plane – strain drawing.

INTRODUCTION

In metal forming processes, fracture often occurs at frictional interfaces. It is important to note that the process of deformation and fracture in a narrow material layer in the vicinity of frictional interfaces is quite different from that in the bulk. This is confirmed by the formation of a material layer with special properties near the frictional interface (see, for example, [1 – 3]). In the case of rigid perfectly plastic materials, the maximum friction surface is defined by the condition that the friction stress is equal to the shear yield stress. A remarkable property of the maximum friction surfaces is that the velocity field is singular in their vicinity [4]. This property can serve as a theoretical basis for describing the aforementioned distribution of material properties and fracture prediction in a narrow layer near the surface. One possible approach has been proposed in [5] where the thickness of the layer of intensive deformation near the friction surface has been evaluated. However, this approach is not appropriate for fracture prediction since it does not involve the hydrostatic stress, whereas the fracture process is significantly influenced by this stress. It has been shown in [4] that the equivalent strain rate defined by $\xi_{eq} = \sqrt{(2/3)\xi_{ij}\xi_{ij}}$, where ξ_{ij} are the components of the strain rate tensor, approaches infinity at the maximum friction surfaces and its behavior follows the law $\xi_{eq} = O\left(1/\sqrt{s}\right)$ where s is the distance from the surface along the normal to it. The equivalent strain rate is involved in many empirical ductile fracture criteria used at large plastic strains, for example [6 – 8]. Obviously, the singular velocity fields lead to the situation where fracture at the maximum friction surfaces occurs at any small level of strain and stress. To overcome this difficulty, it is possible to adopt non – local fracture criteria. Note that the situation described is very similar to that in classical fracture mechanics where fracture conditions in terms of stress predict crack propagation at any level of stress [9]. A non-local fracture criterion in the vicinity of crack tips has been proposed in [10]. The same idea is used in the present paper to predict fracture in the vicinity of friction surfaces. The approach is then adopted to design plane-strain drawing.

FRACTURE CRITERION

A popular empirical fracture criterion used for fracture prediction in metal forming processes is based on the workability diagram proposed in [11]. The workability diagram is a curve in the coordinates $\beta\varepsilon_{eq}$, where ε_{eq} is the equivalent strain, β is the triaxiality factor defined by $\beta = 3\sigma/\sigma_{eq}$, σ is the hydrostatic stress, $\sigma_{eq} = \sqrt{(3/2)s_{ij}s_{ij}}$ is the equivalent stress, and s_{ij} are the deviatoric components of the stress tensor. The fracture criterion is formulated as

$$\varepsilon_{eq}^{f} = \Phi\left(\beta_{av}\right) \quad \text{and} \quad \beta_{av} = \frac{1}{\varepsilon_{eq}}\int_{0}^{t}\beta\xi_{eq}dt \tag{1}$$

where ε_{eq}^{f} is the equivalent strain at fracture and the integration is carried out at the material particle. The function $\Phi\left(\beta_{av}\right)$ should be found from experiment. Fracture does not occur if $\varepsilon_{eq} < \varepsilon_{eq}^{f}$. This inequality is never satisfied at the maximum friction surface since $\varepsilon_{eq} \to \infty$ there. Following [10] it is assumed here that the non-local fracture criterion generalizing the aforementioned local fracture criterion is

$$\varepsilon_{eq} = \varepsilon_{eq}^{f} = \Phi\left(\beta_{av}\right) \quad \text{at} \quad s \le s_c \tag{2}$$

where s is, as before, the distance from the friction surface and s_c is a material parameter. If the criterion (2) is accepted, fracture occurrence depends on process parameters despite of the condition $\varepsilon_{eq} \to \infty$ as $s \to 0$. To the best of our knowledge, there are no experimental data to determine the value of s_c. However, it is known [3] that the thickness of the layer of intensive deformation is about 1/10 of the characteristic length of the process. Therefore, as a first approximation, it is possible to assume that s_c is also equal to 1/10 of the characteristic length of the process.

PLANE – STRAIN DRAWING

In many cases an approximate analysis of the process of drawing and extrusion is based on solutions obtained for flow through infinite channels, for example [12, 13]. In the case of plane – strain drawing through a wedge-shaped die (Fig.1), an appropriate solution has been obtained in [14].

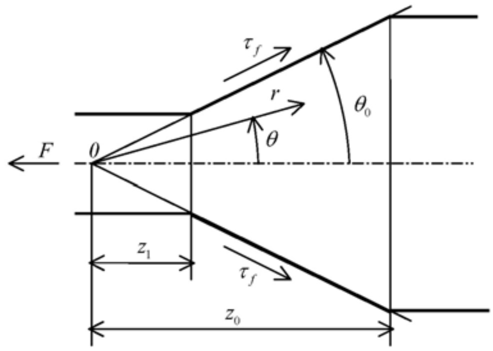

Figure 1. Plane – strain drawing – notation

In particular, in a cylindrical coordinate system $r\theta$ the stress field is given by

$$\frac{\sigma_{rr}}{\tau_s} = \cos 2\psi + \frac{c}{2}\ln\left(\frac{c}{c-2\cos 2\psi}\right) - c\ln\left(\frac{r}{Hr_0}\right),$$
$$\frac{\sigma_{\theta\theta}}{\tau_s} = -\cos 2\psi + \frac{c}{2}\ln\left(\frac{c}{c-2\cos 2\psi}\right) - c\ln\left(\frac{r}{Hr_0}\right), \quad (3)$$
$$\frac{\sigma_{r\theta}}{\tau_s} = \sin 2\psi$$

where σ_{rr}, $\sigma_{\theta\theta}$ and $\sigma_{r\theta}$ are the stress components in the cylindrical coordinates, τ_s is the shear yield stress, c and r_0 are constant, $2H$ is the thickness of the strip at the entrance ($2h$ will stand for the thickness of the strip at the exit), and ψ is a function of θ which is determined by the following equation

$$\theta = -\psi + c\,\mathrm{arctg}\left[\left(\frac{c+2}{c-2}\right)^{1/2}\mathrm{tg}\,\psi\right]\left(c^2-4\right)^{-1/2} \quad (4)$$

The only non-zero component of the velocity vector is u_r and it is given by

$$u_r = -\frac{U_0}{r\left(c-2\cos 2\psi\right)} \quad (5)$$

Let $2\theta_0$ be the die angle. The maximum friction law is assumed at the die surface. Then,

$\psi = \pi/4$ at $\theta = \theta_0$ and the equation for determining *c* follows from (4) in the form

$$\theta_0 = -\frac{\pi}{4} + c\,\mathrm{arctg}\left[\left(\frac{c+2}{c-2}\right)^{1/2}\right]\left(c^2 - 4\right)^{-1/2} \tag{6}$$

Following [12] it is assumed that

$$\int_0^{\theta_0} \sigma_z\big|_{z=z_0} \frac{d\theta}{\cos^2\theta} = 0 \tag{7}$$

where *z* is the distance from point *O* (Fig.1) and σ_z is defined by

$$\sigma_z = \frac{1}{2}\left(\sigma_{rr} + \sigma_{\theta\theta}\right) + \frac{1}{2}\left(\sigma_{rr} - \sigma_{\theta\theta}\right)\cos 2\theta - \sigma_{r\theta}\sin 2\theta \tag{8}$$

Substituting (8), with the use of (3) and (4), into (7) gives the equation for determining r_0.
Using (5) it is possible to find the equivalent strain at each particle when it exits the plastic zone in the following form

$$\varepsilon_{eq}^{ex} = \frac{\sqrt{3}}{\cos 2\psi}\ln\left(\frac{H}{h}\right) \tag{9}$$

Using $(1)^2$, (3), (5) and (9) it is possible to find β_{av} at each particle when it exits the plastic zone in the following form

$$\beta_{av}^{ex} = \sqrt{3}c\ln\left(\frac{c}{c - \cos 2\psi}\right) - \frac{\sqrt{3}c}{\ln\left(H/h\right)}\int_{r_{ex}}^{r_{en}} r^{-1}\ln\left(\frac{r}{Hr_0}\right)dr \tag{10}$$

where r_{en} and r_{ex} are the radial coordinates of the particle when it enters and exits the plastic zone, respectively. It follows from geometrical considerations (Fig.1) that

$$r_{en} = \frac{H\cot\theta_0}{\cos\psi} \quad \text{and} \quad r_{ex} = \frac{h\cot\theta_0}{\cos\psi} \tag{11}$$

According to the fracture criterion proposed, it is necessary to substitute (9) and (10) into $(1)^1$ at $\theta = 0.9\theta_0 = \theta_{cr}$. The corresponding value of ψ, ψ_{cr}, can be found from (4). As a result, the relation between H/h and θ_0 at fracture is obtained.

NUMERICAL EXAMPLE

Following the procedure described in the previous section, numerical calculations have been carried out for steel of chemical composition 0.32-0.39% C; 0.15-0.35% Si; 0.5-0.8% Mn. For this steel the function

$\Phi\left(\beta_{av}\right)$ is [15] $\Phi\left(\beta_{av}\right) = 0.639 - 0.568\beta_{av} + 0.138\beta_{av}^2$ (12)

First, equations (6) and (7) have been solved numerically. Then, the values of c and r_0 obtained for a prescribed value of θ_0 have been substituted into (9) and (10), with the use of (11). Finally, equation $(1)^1$, with the use of (12), has been used to obtain the value of H/h at which fracture occurs. The result of these calculations is illustrated in Fig.2 where the dependence of the reduction (H/h) on the die angle is shown.

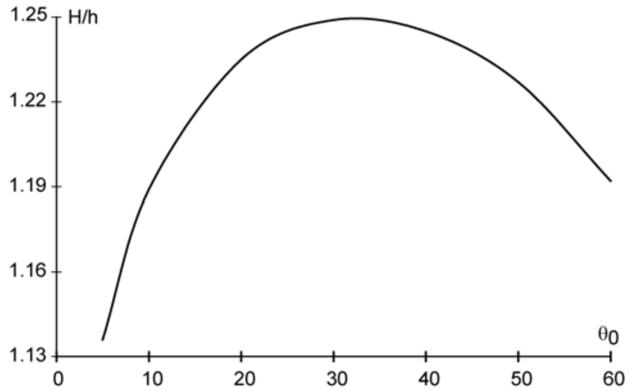

Figure 2. Dependence of the reduction on the die angle at fracture

CONCLUSIONS

A non-local ductile fracture criterion is proposed. The criterion is based on the workability diagram and generalizes a popular local ductile fracture criterion. A necessity to introduce non-local fracture criteria follows from the fact that the velocity field is singular in the vicinity of maximum friction surfaces for several popular models of plasticity. In such cases local fracture criteria cannot be adopted since they predict fracture at the very beginning of the process independently of other conditions. Based on the new fracture criterion, a simplified engineering design of plane-strain drawing has been proposed. It is seen (Fig.2) that the maximum possible reduction corresponds to the die angle of 35^0 (approximately).

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PRIMENA NE LOKALNOG KRITERIJUMA ZA ODREĐIVANJE NASTANKA PUKOTINE U PROCESU RAVANSKOG VUČENJA

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REZIME

Popularna metoda za određivanje nastanka prve pukotine u materijalu za vreme plastičnog deformisanja je bazirana na konceptu dijagrama granične obradivosti. Međutim, za neke konstitutivne modele ova metoda nije moguća ako pukotina nastaje u ravni maksimalnog trenja jer je u tom slučaju polje brzina singularno, tako da se efektivna brzina deformisanja na takvim ravnima približava infinitivnoj.
Da bi se prevazišla ova teškoća moguće je usvojiti kriterijum ne-lokalne pukotine. Taj kriterijum može, takođe, biti baziran na dijagramu granične obradivosti.
U ovom radu takav kriterijum je primenjen za određivanje nastanka prve pukotine u procesu ravanskog vučenja.