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PROPERTIES OF THE STRESS VECTOR IN THE CONDITIONS OF PLANE STRESS

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ABSTRACT

The plane state of stress for some deformable body is analysed in this paper. Besides, linear operator is used as mathematical mean. Liner operator is actually a mixed second-order tensor. However, the application of liner operator allows the analysis of stress with the methods of vector algebra.
It is demonstrated that, because of the symmetry of linear operator in the plane state of stress,the two principal directions of stress exist which linear operator leaves alone. Further, the intensities and the locations of the principal normal stresses are expressed relative to cartesian coordinate system very simple.

Key words: *stress vector, linear operator, plane stress*

1. INTRODUCTION

Some properties of the stress vector are analysed for a deformable body, which there is in the conditions of plane stress. In the case of the space stress can be proved that at anyone point of a deformable medium at least one the principal direction exists always. For the plane stress, in general case, such direction needn't exist. The symmetry of stress tensor renders certain, and in a plane, the existence the principal directions of normal stress.
Some properties of stress vector are analysed through the medium of linear operator. Linear operator is in fact the mixed second-order tensor, and study of his is the same what and studies the mixed second-order tensor. Namely, in the stress theory the state of stress at anyone points of a deformable medium completely is determined by the linear vector function.
Firstly, it is demonstrated that of second-order polynomial noughts are real. After this, the proof is given for the existence and the location of principal normal stresses, which are mutually perpendicular. Finite, it is allowed the calculation of the algebraic value of the principal normal stresses.

2. STRESS VECTOR IN THE PLANE

For the state of stress of the deformable body is said that is plane (two-dimensional), if at every point, for the everything planes which can be placed throughout every of points of this body, the stress vector lies in one end same the plane. In other words, the state of stress of the body is plane if and alone if the coordinate stress vectors $\vec{p}_1, \vec{p}_2, \vec{p}_3$ are complainer, for every point and every plane, i.e.,

$$\left(\vec{p}_1 \times \vec{p}_2\right) \cdot \vec{p}_3 = 0 . \tag{1}$$

Let the coordinate vector $\vec{p}_3$ is dependent of the coordinate vectors $\vec{p}_1, \vec{p}_2$.
For the base $\left(\vec{e}_1, \vec{e}_2\right)$ in the vector space $\mathbf{X}_0$ (M) end anyone two vectors $\vec{p}_1, \vec{p}_2$, from the vector space $\mathbf{X}_0$ (M), one and same one linear operator $A : \mathbf{X}_0(M) \to \mathbf{X}_0(M)$ exists such that is

$$A\vec{e}_1 = \vec{p}_1, A\vec{e}_2 = \vec{p}_2 . \tag{2}$$

Every vector $\vec{p}_i = A\vec{e}_i \in \mathbf{X}_0 (M)$ has unique presentation in the base $\left(\vec{e}_1, \vec{e}_2\right)$. Therefore is

$$\left.\begin{aligned} \vec{p}_1 &= A\vec{e}_1 = \sigma_{11}\vec{e}_1 + \sigma_{21}\vec{e}_2 \\ \vec{p}_2 &= A\vec{e}_2 = \sigma_{12}\vec{e}_1 + \sigma_{22}\vec{e}_{22} . \end{aligned}\right\} \tag{3}$$

Here by the operator A and the base $\left(\vec{e}_1, \vec{e}_2\right)$ are determined complete the scalars σ_{ij} $(i,j = 1,2)$. The stress vector $\vec{p}$ now can be expressed in the form

$$\vec{p} = \sum_{i=1}^{2} \sum_{j=1}^{2} a_i \sigma_{ij} \vec{e}_i , \tag{4}$$

where a_i are the cosines direction of the normal $\vec{n}$ the plane in which acting the stress vector $\vec{p}$.
By using the characteristics

$$\vec{e}_1^{\,'} = a_1\vec{e}_1, \vec{e}_2^{\,'} = a_2\vec{e}_2$$

to (4), is obtained equation

$$\vec{p} = \sum_{i=1}^{2} \sum_{j=1}^{2} \sigma_{ij} \vec{e}_i^{\,'} , \tag{5}$$

which with (3) gives

$$\vec{p} = A\vec{e}_1^{\,'} + A\vec{e}_2^{\,'} = A(\vec{e}_1^{\,'} + \vec{e}_2^{\,'}) = A\vec{n} \tag{6}$$

the stress vectors $\vec{p}$ as the result acting of linear operator A on normal of the plane.

3. EXISTENCE PRINCIPAL DIRECTIONS OF NORMAL STRESSES

Because of the conjugation of the shear stress $\sigma_{ij} = \sigma_{ji}$ $(i \neq j)$, the matrix

$$\mathbf{A} = \begin{vmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{vmatrix} \tag{7}$$

of the linear operator A is symmetrical. The symmetry of the matrix $\mathbf{A}$ of the linear operator A withdraws and the symmetry of the linear operator A. For operator A is said that is symmetrical as is

$$A\,\mathbf{x} \cdot \mathbf{y} = \mathbf{x} \cdot A\,\mathbf{y} \tag{8}$$

for everything the $\mathbf{x}, \mathbf{y} \in \mathbf{X}_0$.

For symmetrical operator $A : \mathbf{X}_0 \to \mathbf{X}_0$ the real numbers σ_1 and σ_2 and the orthonormal base $(\mathbf{e}_1, \mathbf{e}_2)$ of the space $\mathbf{X}_0$ exist such that is

$$A\,\mathbf{e}_1 = \sigma_1\,\mathbf{e}_1, \quad A\,\mathbf{e}_2 = \sigma_2\,\mathbf{e}_2 \,. \tag{9}$$

For the proof of the relations (9), beside the symmetry of linear operator A, is used in fact that in the space $\mathbf{X}_0$ are every tree vectors linear dependent and that two linear independent vectors exist, what different is said that is the $\mathbf{X}_0$ two-dimensional vector space. Separately between the vectors $\mathbf{e}$, $A\,\mathbf{e}$, $A^2\,\mathbf{e}$ exists linear dependent for every vector $\mathbf{e}$, $A\,\mathbf{e}$, $A^2\,\mathbf{e}$. If $\mathbf{e} \neq \mathbf{0}$ is. Then two possibilities exist.

1.The vector $\mathbf{e}$ and $A\,\mathbf{e}$ are linear dependent. The real number σ_1 exists such that is $A\,\mathbf{e}_1 = \sigma_1\,\mathbf{e}$. The vector $\mathbf{e}_1 = \mathrm{e}/|\mathrm{e}|$ is unit vector and $A\mathbf{e}_1 = \sigma_1\,\mathbf{e}_1$. If $\mathbf{e}_2$ is unit vector, which is perpendicular to the $\mathbf{e}_1$, then is

$$\mathbf{e}_1 \cdot A\,\mathbf{e}_2 = A\,\mathbf{e}_1 \cdot \mathbf{e}_2 = (\sigma_1\,\mathbf{e}_1\,) \cdot \mathbf{e}_2 = \sigma_1\,(\mathbf{e}_1 \cdot \mathbf{e}_2\,) = 0.$$

Since the vector $A\,\mathbf{e}_2$ is perpendicular to the $\mathbf{e}_1$, he must be collinear with the $\mathbf{e}_2$; well the real number σ_2 exists such that is $A\,\mathbf{e}_2 = \sigma_2\,\mathbf{e}_2$. In this case the relations (9) are proved.

2. The vector $\mathbf{e}$ and $A\,\mathbf{e}$ are linear independent. Because of the dependent of the vectors $\mathbf{e}$, $A\,\mathbf{e}$, $A^2\,\mathbf{e}$ in the plane, the real number α and β exist such that is $A^2\,\mathbf{e} = \alpha\,A\,\mathbf{e} + \beta\,\mathbf{e}$, i.e.

$$P\,(A\,)\mathbf{e} = \mathbf{0}, \tag{10}$$

where

$$P\,(\lambda\,) = \lambda^2 - \alpha\,\lambda - \beta \tag{11}$$

is second-order polynomial with the real coefficients.

If the supposition about the symmetry of the linear operator A is used, it is proved now that the noughts of polynomial (11) are real, and this is the central part of the proof of the relation (9). If it is supposed contrary, that $b + i\,t$ $(t \neq 0)$ $(b, t \in \mathbf{R})$ is nought of polynomial (11), i.e. that is

$(b + i\,t)^2 - \alpha\,(b + i\,t) - \beta = 0$. From here, with crossing to the conjugate complex numbers, is given $(b - i\,t)^2 - \alpha\,(b - i\,t) - \beta = 0$, i.e. and the number $b - i\,t$ is nought of the polynomial (11), now

$$P(\lambda) = [\lambda - (b + i\,t)][\lambda - (b - i\,t)] = (\lambda - b)^2 + t^2,$$

together with (10) gives

$$(A - b\,I)^2 + t^2\,\mathbf{e} = \mathbf{0}. \tag{12}$$

The operator $A - b\,I$ is symmetrical, this can be seen from $(A - \sigma I)\,\mathbf{x} \cdot \mathbf{y} = (A\,\mathbf{x} - b\,\mathbf{x}) \cdot \mathbf{y} = A\,\mathbf{x} \cdot \mathbf{y} - b\,\mathbf{x} \cdot \mathbf{y} = A\,\mathbf{x} \cdot \mathbf{y} - b\,\mathbf{x} \cdot \mathbf{y} = \mathbf{x} \cdot A\,\mathbf{y} - \mathbf{x} \cdot b\,\mathbf{y} = \mathbf{x} \cdot (A - b\,I)\,\mathbf{y}$.

If (12) is multiplies scalar with the $\mathbf{e}$, then because of

$$(A - b\,I)^2\,\mathbf{e} \cdot \mathbf{e} = (A - b\,I)\,(A - b\,I)\,\mathbf{e} \cdot \mathbf{e} = (A - b\,I)\,\mathbf{e} \cdot (A - b\,I)\,\mathbf{e} = |(A - b\,I)\,\mathbf{e}|^2$$

is obtained

$$|(A - \sigma I)\,\mathbf{e}|^2 + t^2\,|\mathbf{e}|^2 = 0. \tag{13}$$

From (13) is obtained that is $|(A - b\,I)\,\mathbf{e}| = 0$ and $t^2\,|\mathbf{e}| = 0$. How is $\mathbf{e} \neq \mathbf{0}$, then is $t = 0$ and this is contrary to the supposition that is $t \neq 0$. In this way is commode to the contradiction. So was proved that polynomial (11) has only real noughts.

Let σ_1 and σ_2 are nought of polynomial (11). It is affirmed that is $\sigma_1 \neq \sigma_2$. Of course $\sigma_1 = \sigma_2$ would pull $P(\lambda) = (\lambda - \sigma_1)^2$, (10) would go over to in $(A - \sigma_1\,I)^2\,\mathbf{e} = \mathbf{0}$. From here the multiplication with the $\mathbf{e}$ is obtained $|(A - \sigma_1\,I)\,\mathbf{e}| = 0$, and this is given $A\,\mathbf{e} - \sigma_1\,\mathbf{e} = \mathbf{0}$. This is contradictory with the supposition that are $\mathbf{e}$ and $A\,\mathbf{e}$ the independent vectors. Well is $P(\lambda) = (\lambda - \sigma_1)(\lambda - \sigma_2)$ with $\sigma_1 \neq \sigma_2$, then (10) pulls

$$(A - \sigma_1\,I)\,(A - \sigma_2\,I)\,\mathbf{e} = \mathbf{0}. \tag{14}$$

If is putted

$$\mathbf{v}_1 = (A - \sigma_1\,I)\,\mathbf{e} = A\,\mathbf{e} - \sigma_2\,\mathbf{e}, \quad \mathbf{v}_2 = (A - \sigma_1\,I)\,\mathbf{e} = A\,\mathbf{e} - \sigma_1\,\mathbf{e}.$$

At that time (14) pulls $(A - \sigma_1\,I)\,\mathbf{v}_1 = \mathbf{0}$ and $(A - \sigma_2\,I)\,\mathbf{v}_2 = \mathbf{0}$, i.e.

$$A\,\mathbf{v}_1 = \sigma_1\,\mathbf{v}_1, \quad A\,\mathbf{v}_2 = \sigma_2\,\mathbf{v}_2. \tag{15}$$

Since $\mathbf{v}_1 \neq \mathbf{0}$ and $\mathbf{v}_2 \neq \mathbf{0}$, because of the independent of the vectors $\mathbf{e}$ and $A\,\mathbf{e}$, the vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are eigenvectors of operator A which corresponding to the eigenvalues σ_1 and σ_2 respectively.

The perpendicularly of the vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ is obtained from (15) such that the first equality is multiplied with $\mathbf{v}_2$, the second with $\mathbf{v}_1$ and the results are deducted. In this way is obtained

$$A\,\mathbf{v}_1 \cdot \mathbf{v}_2 - A\,\mathbf{v}_2 \cdot \mathbf{v}_1 = \sigma_1\,\mathbf{v}_1 \cdot \mathbf{v}_2 - \sigma_2\,\mathbf{v}_2 \cdot \mathbf{v}_1. \tag{16}$$

Because of the symmetry of operator A the left side vanishes in (16), the right side goes over to $(\sigma_1 - \sigma_2)\,\mathbf{v}_1 \cdot \mathbf{v}_2 = 0$, what because of $\sigma_1 \neq \sigma_2$ given $\mathbf{v}_1 \cdot \mathbf{v}_2 = 0$. If $\mathbf{e}_1 = \mathrm{v}_1/|\mathrm{v}_1|$ and $\mathbf{e}_2 = \mathrm{v}_2/|\mathrm{v}_2|$ is putted, it is obtained the orthonormal base $(\mathbf{e}_1, \mathbf{e}_2)$ of the space $\mathbf{X}_0$ for which (9) is worth.

4. INTENSITY AND LOCATION OF PRINCIPAL NORMAL STRESSES

The intensity of the normal component σ_n of the total stress $\vec{p}$ is obtained scalar product of the stress vector (6) with the unit vector $\vec{n}$ of the observed plane, i.e.

$$\sigma_n = A\,\mathbf{n}\cdot\mathbf{n} = A\,(\cos\alpha_1\,\mathbf{e}_1 + \cos\alpha_2\,\mathbf{e}_2)\cdot(\cos\alpha_1\,\mathbf{e}_1 + \cos\alpha_2\,\mathbf{e}_2)$$
$$= (\cos\alpha_1\,A\,\mathbf{e}_1 + \cos\alpha_2\,A\,\mathbf{e}_2)\cdot(\cos\alpha_1\,\mathbf{e}_1 + \cos\alpha_2\,\mathbf{e}_2) \Rightarrow$$

$$\sigma_n = \cos\alpha_1\,(\sigma_{11}\,\mathbf{e}_1 + \sigma_{21}\,\mathbf{e}_2)\cdot(\cos\alpha_1\,\mathbf{e}_1 + \cos\alpha_2\,\mathbf{e}_2)$$
$$+ \cos\alpha_2\,(\sigma_{12}\,\mathbf{e}_1 + \sigma_{22}\,\mathbf{e}_2)\cdot(\cos\alpha_1\,\mathbf{e}_1 + \cos\alpha_2\,\mathbf{e}_2)$$

or written down short

$$\sigma_n = (\sum_{i=1}^{2} x_i \sum_{j=1}^{2} a_{ji}\vec{e}_j)\cdot(\sum_{j=1}^{2} x_j\vec{e}_j) = \sum_{i,j=1}^{2} \sigma_{ji}x_i x_j \qquad (\sigma_{ij} = \sigma_{ji}), \tag{17}$$

where the characteristic $x_i = \cos\,\alpha_i$ (i =1,2) are introduced. The function (polynomial)

$$\sigma_n(x_1, x_2) = \sum_{i,j=1}^{2} \sigma_{ji}x_i x_j = \sigma_{11}\,x_1^2 + 2\,\sigma_{12}\,x_1\,x_2 + \sigma_{22}\,x_2^2 \tag{18}$$

is named quadratic form of two variables. For the matrix

$$\mathbf{A} = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} \tag{19}$$

is said that the matrix is of form (18). Linear operator $A : \mathbf{X}_0 \rightarrow \mathbf{X}_0$ which corresponding to the matrix (19) is symmetrical. For symmetrical linear operator A the right orthonormal base $(\vec{f}_1, \vec{f}_2)$ exists and the real numbers σ_1 and σ_2 such that is

$$A\,\mathbf{f}_1 = \sigma_1\,\mathbf{f}_1, \quad A\,\mathbf{f}_2 = \sigma_2\,\mathbf{f}_2. \tag{20}$$

The base $(\vec{f}_1, \vec{f}_2)$ emerges from the base ($\mathbf{e}_1$, $\mathbf{e}_2$) with the rotation for the angle φ for which is $\cos\varphi = \mathbf{f}_1\cdot\mathbf{e}_1$. If $\mathbf{x} = x_1\,\mathbf{e}_1 + x_2\,\mathbf{e}_2 = x_1'\,\mathbf{f}_1 + x_2'\,\mathbf{f}_2$ is anyone vector from the $\mathbf{X}_0$, then is

$$x_1' = x_1\cos\varphi + x_2\sin\varphi, \quad x_2' = -\,x_1\sin\varphi + x_2\cos\varphi\,. \tag{21}$$

Since

$$A\,\mathbf{n}\cdot\mathbf{n} = (x_1'\,A\,\mathbf{f}_1 + x_2'\,A\,\mathbf{f}_2)\cdot(x_1'\,\mathbf{f}_1 + x_2'\,\mathbf{f}_2)\cdot\sigma_1\,(x_1')^2 + \sigma_2\,(x_2')^2,$$

it is fended that worth

$$\sigma_{11}\,x_1^2 + 2\sigma_{12}\,x_1\,x_2 + \sigma_{22}\,x_2^2 = \sigma_1\,(x_1\cos\varphi + x_2\sin\varphi)^2 + \sigma_2\,(-\,x_1\sin\varphi + x_2\cos\varphi)^2 \tag{22}$$

for all the real number x_1 and x_2. The compassion of the polynomial coefficients on the left side in (22) with coefficients, which are obtained after squarer on the right side in (22), is obtained

$$\left.\begin{aligned} \sigma_1 \cos^2\varphi + \sigma_2 \sin^2\varphi &= \sigma_{11} \\ \sigma_1 \sin^2\varphi + \sigma_2 \cos^2\varphi &= \sigma_{22} \\ (\sigma_1 - \sigma_2)\sin\varphi\cos\varphi &= \sigma_{12}. \end{aligned}\right\} \tag{23}$$

From (23) is obtained

$$\det \mathbf{A} = \sigma_{11}\,\sigma_{22} - \sigma_{12}^2 = [\sigma_1^2 + \sigma_2^2 - (\sigma_1 - \sigma_2)^2]\sin^2\varphi\cos^2\varphi + \sigma_1\,\sigma_2(\cos^4\varphi + \sin^4\varphi) = \sigma_1\,\sigma_2(\cos^2\varphi + \sin^2\varphi)^2 \Rightarrow$$

$$\det \mathbf{A} = \sigma_{11}\,\sigma_{22} - \sigma_{12}^2 = \sigma_1\,\sigma_2. \tag{24}$$

Adding the first two equation in (23) gives

$$\mathrm{tr}\,\mathbf{A} = \sigma_{11} + \sigma_{22} = \sigma_1 + \sigma_2, \tag{25}$$

where with tr **A** is denoted the addition diagonal elements of the matrix **A**. This number is cooled the trace of the matrix **A**. From (24) and (25) it is seen that are the eigenvalues σ_1 and σ_2 of operator A nought of polynomial $\sigma^2 - \mathrm{tr}\,\mathbf{A} + \det\mathbf{A}$; then can be taken that is

$$\left.\begin{aligned} \sigma_1 &= \frac{1}{2}\{\mathrm{tr}\,\mathbf{A} - [(\mathrm{tr}\,\mathbf{A})^2 - 4\det\mathbf{A}]^{1/2}\}, \\ \sigma_2 &= \frac{1}{2}\{\mathrm{tr}\,\mathbf{A} - [(\mathrm{tr}\,\mathbf{A})^2 + 4\det\mathbf{A}]^{1/2}\}. \end{aligned}\right\} \tag{26}$$

In this place can be descried that is $(\mathrm{tr}\,\mathbf{A})^2 - 4\det\mathbf{A} = (\sigma_{11} - \sigma_{22})^2 + 4\quad\sigma_{12}^2$.

If $\sigma_{12} \neq 0$, then with the deduction the second equation of the first equation in (23) is obtained $(\sigma_1 - \sigma_2)\cos 2\varphi = \sigma_{11} - \sigma_{22}$, what together with the third equation $(\sigma_1 - \sigma_2)\sin 2\varphi = 2\,\sigma_{12}$ from (23) leads on

$$ctg\,2\varphi = \frac{\sigma_{11} - \sigma_{22}}{2\sigma_{12}}. \tag{27}$$

From (27) is calculated the angle φ for which the base ($\mathbf{e}_1$, $\mathbf{e}_2$) ought to rotate, and from (26) the numbers such that (22) is worth.

5. CONCLUSION

Analysis the state of stress is considerable simplified by using of linear operator. The methods of vector algebra is used here for the study the state of stress at a point in the place usual the study by methods of tensor analysis. In this way complete proceeding is approached a broad circles of interested party. On the other hand, linear operator in fact is mixed second-order tensor.
In the case of the space stress at everyone point of the deformable media, the principal direction exists, always, which linear operator leaves alone. Such direction needn't exists at the plane stress, in general case. The symmetry of the stress tensor renders and in a plane the existence the principal directions of the normal stress. For the proof of this assertion, save the symmetry of linear operator, as indispensable fact is used that in the two-dimensional vector space are anyone tree vectors linear dependent.
At all events, these facts are well known. Meanwhile, here is used the original access to this problem.

6. REFERENCE

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OSOBINE VEKTORA NAPONA U USLOVIMA RAVNINSKIH NAPONA

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REZIME

U radu se analizira ravninsko naponsko stanje. U slučaju prostornog naponskog stanja, može se dokazati da u bilo kojoj tački deformabilne sredine uvijek postoji bar jedan glavni pravac[1]. U opštem slučaju, kod ravninskog naponskog stanja takav pravac nemora postojati.Simetričnost linearnog operatora (tenzora napona) obezbjeđuje i u ravnini postojanje glavnih pravaca normalnih napona. U dokazivanju navedene tvrdnje, osim simetričnosti linearnog operatora, kao bitna činjenica koristi se da su u dvodimenzionalnom prostoru svaka tri vektora linearno zavisna.

Analiza osobina vektora napona ovdje se provodi posredstvom linearnog operatora umjesto uobičajene analize posredstvom tenzora napona. Međutim, linearan operator je ustvari mješoviti tenzor drugog ranga i njegovo izučavanje je isto što i izučavanje mješovitog tenzora drugog ranga. Ovaj pristup omogućava analizu naponskog stanja metodama linearne algebre što, zbog jednostavnosti pristupa, omogućava praćenje analize širem krugu zainteresovanih.

U teoriji stanja napona, stanje napona u bilo kojoj tački deformabilne sredine u potpunosti je određeno linearnom vektorskom funkcijom. Uobičajeno je za funkciju koja ima domenu i kodomenu u vektorskom prostoru zvati operatorm.

Najprije se pokazuje da su nule polinoma drugog reda realne. Poslije toga daju se dokazi o postojanosti i položaju glavnih pravaca normalnih napona koji su međusobno upravni. Konačno se omogućava, na veoma jednostavan način, izračunavanje vrijednosti glavnih normalnih napona.

Svakoko, navedene činjenice su poznate. Međutim, ovdje se radi o originalnom pristupu ovoj problematici.