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FLOW OF METAL DURING EXTRUSION: THREE-DIMENSIONAL SIMULATION BY FINITE VOLUME METHOD

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ABSTRACT

A finite volume method for prediction of plastic flows of metal during extrusion processes is described. The method uses discretisation elements of an arbitrary polyhedral shape, colocated variable arrangement, and is based on the SIMPLE algorithm. After discretisation of the governing equations, the resulting system of nonlinear algebraic equations is solved by an iterative procedure, using a segregated algorithm approach. Here it is used for 3D simulation of an extrusion process, showing a very good agreement with experimental and FEM numerical results.
Key words*: Finite Volume Method, Extrusion, 3D modelling*

1. INTRODUCTION

At the present time, design of extrusion dies and operation is primarily based on trial and error. The experience of the die designer, the press operator and the die corrector to a large extent determine the performance of the process. In order to improve the performance, it is necessary to have more information about the extrusion process. Numerical simulations can be a valuable tool in obtaining it [1,6-8]. In this paper, the finite volume method for 3D predictions of metal flow during extrusion is presented. The finite volume method has dominated computational fluid dynamics for many years and has recently emerged as a viable numerical method for stress analysis in solid structures [4,5].

2. MATHEMATICAL MODEL

The process of cold extrusion is governed by the following momentum and mass balance equations:

$$\int_S \rho\mathbf{v}\otimes\mathbf{v}\cdot\mathbf{n}dS = \int_S \boldsymbol{\sigma}\cdot\mathbf{n}dS\,, \tag{1}$$

$$\int_S \rho \mathbf{v} \cdot \mathbf{n} dS = 0 \,, \tag{2}$$

which are valid for an arbitrary part of continuum of the volume V bounded by the surface S, with surface vector $\mathbf{n}$ pointing outwards. In equations (1) and (2) ρ represents the density, $\mathbf{v}$ is the velocity vector and $\boldsymbol{\sigma}$ is the stress tensor.

In the simulation of extrusion processes, it is common to use a rigid-(visco)-plastic constitutive model to describe the material behavior and thus neglect the elastic properties of material. The reason for this is that the elastic deformations are small compared to the very large plastic deformations that occur during the process. For the relation between the stress tensor and the strain-rate tensor, the Levy-Mises equations are used:

$$\boldsymbol{\sigma} = -p\mathbf{I} + 2\mu\dot{\boldsymbol{\varepsilon}} \,, \tag{3}$$

where $p = 1/3 \; tr\boldsymbol{\sigma}$ is the pressure, $\dot{\boldsymbol{\varepsilon}}$ is the strain rate tensor, and the viscosity μ is defined as

$$\mu = \frac{1}{3}\frac{\overline{\sigma}}{\overline{\dot{\varepsilon}}} \,, \tag{4}$$

where $\overline{\sigma} = 3/2\,(\boldsymbol{\sigma}^d : \boldsymbol{\sigma}^d)^{1/2}$ is the effective stress and $\overline{\dot{\varepsilon}} = 2/3\,(\dot{\boldsymbol{\varepsilon}} : \dot{\boldsymbol{\varepsilon}})^{1/2}$ is effective strain rate.
Too small values of effective strain rate in equation (4) may cause the numerical instability, and the limiting strain-rate $\overline{\dot{\varepsilon}}_0$, under which the material is considered to be rigid, must be introduced into calculation. One suggestion for determination of $\overline{\dot{\varepsilon}}_0$ is given in [2,3,10].

In addition to the governing and constitutive equations, appropriate boundary conditions have to be specified at the boundaries of the solution domain. They can be of Dirichlet (velocity prescribed, e.g. ram speed), or Neumann (e.g. zero gradient of velocity in extrusion direction on the die exit) boundary conditions. The Coulomb's friction model is used to specify boundary conditions at the contact boundaries [2].

3. NUMERICAL METHOD

3.1. FINITE VOLUME DISCRETISATION

Equations (1) and (2) are discretised by employing a finite volume discretisation as described by Demirdžić and Muzaferija [5]. The spatial domain is discretised into a finite number of contiguous arbitrarily shaped control volumes (CV) of volume V bounded by cell faces S_j, with computational nodes placed in the centre of each CV. The boundary nodes, necessary for the specification of boundary conditions, are located in the centre of boundary cell faces, Figure 1.

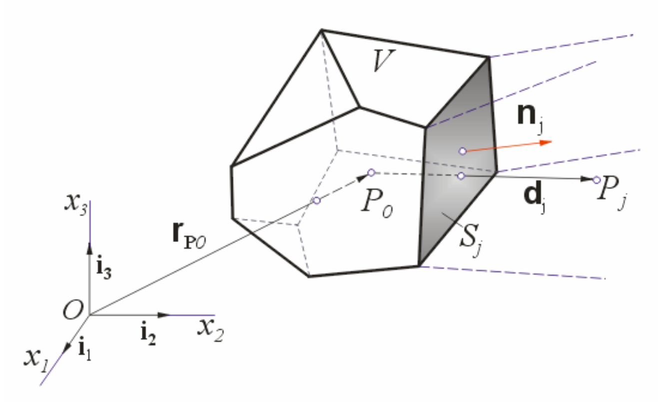


Fig. 1. Control volume of an arbitrary polyhedral shape

Introducing the constitutive relation (3), equation for momentum balance (1) can be integrated over each CV:

$$\sum_{j=1}^{n}\int_{S_j}\rho v_i \mathbf{v}\cdot\mathbf{n}dS-\sum_{j=1}^{n}\int_{S_j}\mu\,\mathrm{grad}\,v_i\cdot\mathbf{n}dS=\sum_{j=1}^{n}\int_{S_j}\left\{\left[\mu(\mathrm{grad}\,v_i)^T+p\mathbf{I}\right]\cdot\mathbf{i}_i\right\}\cdot\mathbf{n}dS \quad (i=1,2,3) \qquad (5)$$

where n is the number of cells which share cell-faces with cell P_0, v_i (i=1,2,3) are the Cartesian velocity vector components and $\mathbf{i}_i$ (i=1,2,3) are the Cartesian base vectors. This equation has three distinct parts: convection and diffusion on the left hand side, and source term on the right hand side. The source term consists of terms coming from the stress tensor that are not contained in the diffusion term on the left hand side.

The gradients in equation (5) are calculated by assuming a linear spatial variation of dependent variables, while the integrals are approximated using the mid-point formula [5]. In this way equation (5) becomes a non-linear algebraic equation for each CV, which links the values of v_i at cell P_0 and its n nearest neighbour cells P_j:

$$a_{v_i 0}v_{i\,P_0}-\sum_{j=1}^{n}a_{v_i\,k}v_{i\,P_j}=b_{v_i}\ . \qquad (6)$$

The coefficient matrix obtained by this procedure is diagonally dominant and sparse with a number of non-zero elements in each raw equal to the number of nearest neighbours n.

The pressure featuring in the source term of the discretised momentum equation is unknown and an independent use of the continuity equation is not made yet. The pressure does not feature explicitly in continuity equation, therefore, some means for coupling the momentum and continuity equation and determining the pressure field are required. This is achieved by employing the predictor-corrector procedure defined by the SIMPLE algorithm [9], which results in a system of linear algebraic equations for pressure correction that has the same form as equation (6).

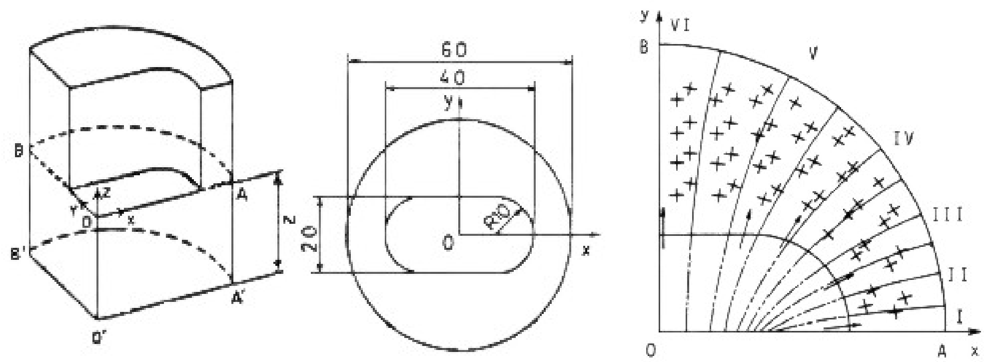
3.2. SOLUTION PROCEDURE

A set of 4*N* non-linear coupled algebraic equations of the form (6) with 4*N* unknowns is obtained for the solution domain consisting of *N* CVs. For the start of calculation, all dependent variables are given the corresponding initial values. Then the boundary conditions are applied. The equations are linearised and de-coupled by assuming that the coefficients and source terms are known and are obtained from the currently available values.
First, linearised momentum equations are solved, by using a conjugate gradient linear equations solver. Then, the pressure-correction equation is solved and the calculated values are used to correct mass fluxes, velocity components and pressure. Finally the coefficient and source terms are updated and the procedure is repeated until a converged solution is obtained.

4. SIMULATION OF EXTRUSION PROCESS

The presented numerical method is applied to a non-symmetric backward extrusion, as shown in Figure 2a, for which experimental and FEM results are given in [6]. An experiment on extrusion is made by the use of a layered plasticine billet and strain distribution is measured on the cross section at the exit. After that, the principal strains are obtained in that cross section and a number of curves along the principal direction are obtained. Then stream surfaces, containing these curves respectively, are determined, with which the deforming region is devided into thin sub-regions, Figure 2b. Shear stress alomost vanishes on these interfaces, because the total shear strain is zero here [6].

Extrusion is done at 10^0C, because plasticine sticks to a punch at room temperature. Extrusion speed is 5 mm/min. Powder of sodium carbonate on the top of a punch and vaselin on a container and on the side surface of a punch are used as a lubricant. Several representative 2D layers are chosen and analyzed by FEM as 2D problems. Material is assumed to be rigid-plastic, with the yield stress σ_Y=0,24 MPa. Maximum friction (sticking) is prescribed on the top of the punch, and zero friction at the container and the side surface of a punch. The 3D solution is obtained by combining these 2D solutions [6].



a) b)

Fig. 2. Problem description [6]

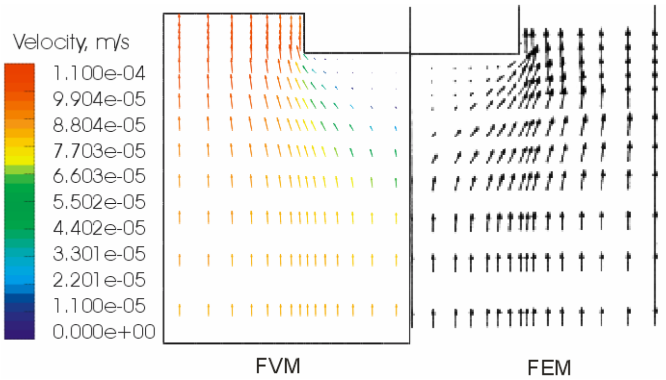


Fig. 3. Velocity distribution for layer II

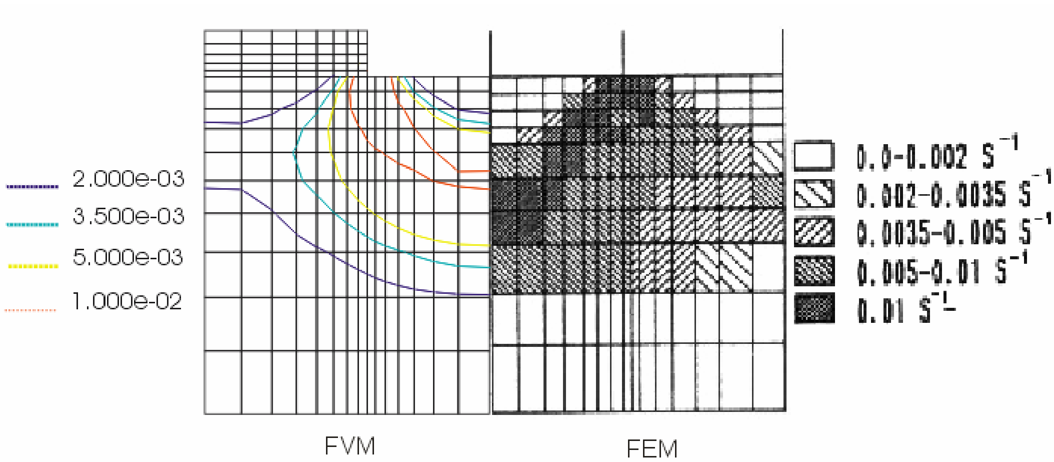


Fig. 4. Pressure distribution for layer II

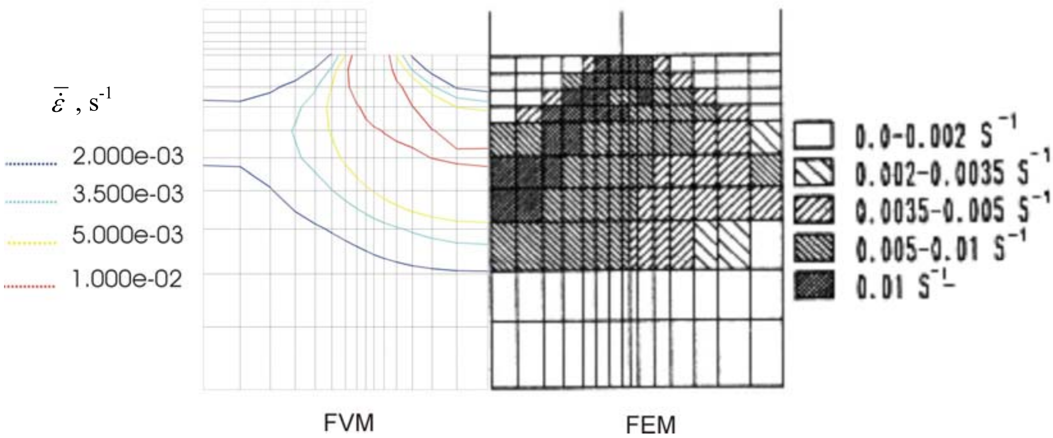
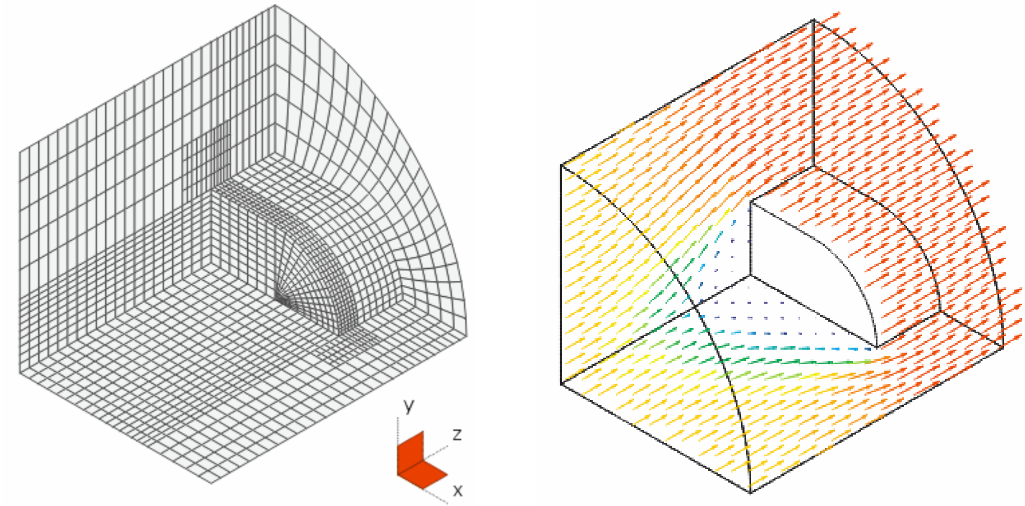


Fig. 5. The distribution of effective strain rate for the layer II

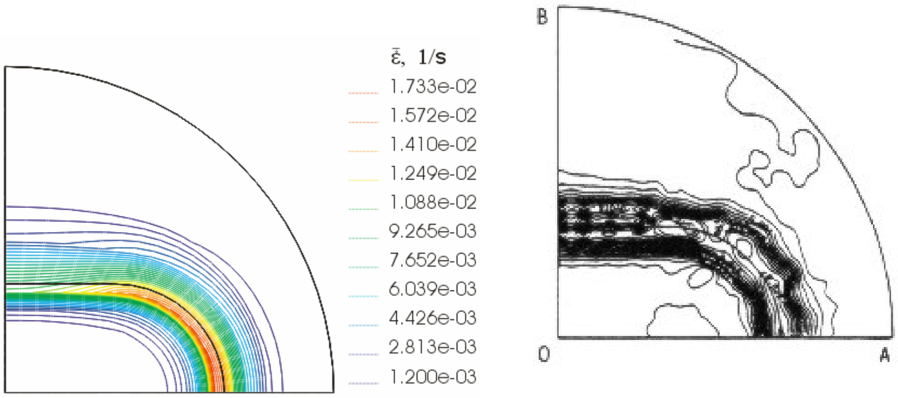
In this paper 2D analysis is done only for layer II, Figure 2b. The obtained velocity, pressure and the distribution of effective strain rate, Figures 3, 4 and 5 respectively, are in good agreement with results obtained by FEM.

The 3D calculation by FVM is done on numerical mesh with 8848 CVs, Figure 6.a, with limiting value of effective strain-rate $\bar{\dot{\varepsilon}}_0 = 1\times10^{-3}\,\mathrm{s}^{-1}$. The resulting velocity field is given on Figure 6.b.

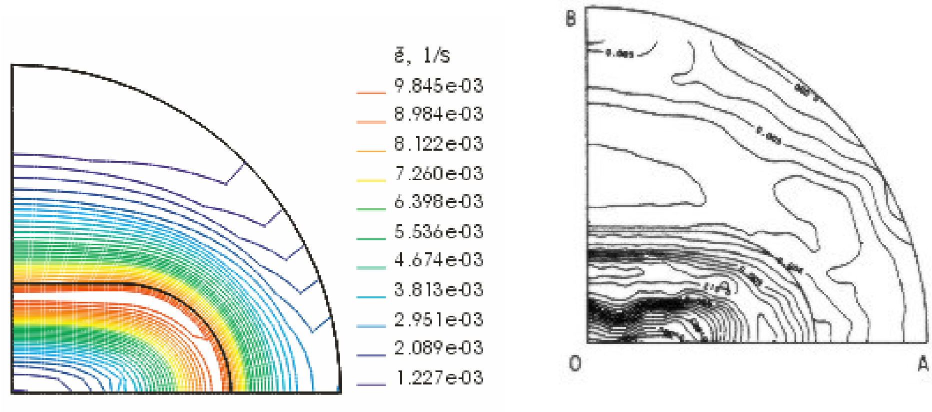


(a) (b)

Fig. 6. Numerical mesh (a) and resulting velocity field (b)



(a) $z = -0{,}55$



(b) $z = -5{,}50$

Fig. 7. Distribution of the effective strain-rate in planes, (a) $z = -0{,}55$ *mm, (b)* $z = -5{,}50$ *mm*

Figures 7a and 7b shows the distribution of the effective strain rate in the planes $z = -0{,}55$ mm and $z = -5{,}50$ mm respectively. The results calculated and presented by Kato et al. [6] are given on the left hand side, and this distributions are in a good correlation with results that are obtained using FVM, Figure 7, the right hand side. Some differences that are exist on Figure 7 are due to introduced approximation about 2D layers, and due to described procedure for obtaining 3D results [6].

5. CONCLUSION

In the work here presented the finite volume method is used to solve the equations governing extrusion, and the distribution of velocity components and velocity and pressure fields throughout the solution domain are calculated, from which the other relevant quantities (e.g. strain-rate and stress tensor components) can easily be obtained. The main advantages of the FVM are its simplicity, and an efficient use of computer resources (steaming out of the iterative segregated solution procedure). The calculated results are in good correlation with experimentally obtained. The further development of the FVM in the analysis of plastic metal flow will concentrate on calculations of the hot extrusion processes and extrusion of viscoplastic materials.

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TOK MATERIJALA ZA VREME PROCESA ISTISKIVANJA I 3D SIMULACIJA METODOM KONAČNIH ELEMENATA

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REZIME

Projektovanje alata i procesa istiskivanja i danas se pretežno obavlja "trial and error" metodom. Iskustvo konstruktora igra dominantnu ulogu u tom procesu.
U savremenim uslovima proizvodnje takav način kreiranja elementata obradnog sistema ne zadovoljava sve strožije zahteve. Jedna od savremenih metoda za suštinsko efikasniju konstrukciju alata je numerička simulacija procesa.
U ovom radu primenjena je 3D simulacija procesa istiskivanja na bazi metode konačnih zapremina (finite volume). Dat je opis metode konačnih zapremina U ovoj metodi koristi se diskretizacija zapremine pomoću elementata polyhedralnog oblika, uz korišćenje SIMPLE algoritma. Nakon diskretizacije nelinearni sistem algebarskih jednačina se rešava iterativnim postupkom pomoću segregatnog pristupa. Ova metoda je godinama bila dominantna u simulacijama problema iz dinamike fluida a u novije vreme se uspešno primenjuje i u mehanici čvrstog tela.
Simulacija je izvršena za proces nesimetričnog suprotnosmernog istiskivanja. Za ovaj proces u jednom ranijem radu [6] dati su rezultati FE analize. Takođe je izvršen i eksperiment procesa istiskivanja korišćenjem plastelina. Tom prilikom detektovano je polje deformacija u meridijalnoj ravni radnog komada. Eksperimentalna istraživanja su izvršena na temperaturi od 10^oC jer na sobnoj temperaturi dolazi do lepljenja plastelina na alat. Kao sredstvo za podmazivanje korišćen je vazelin i sodium carbonat. U FE analizi materijal je smatran kruto-plastičnim, sa naponom tečenja σ_y=0,24 Mpa.
3D simulacija metodom konačnih zapremina i FE simulacija pokazali su dobro slaganje sa eksperimentalnim rezultatima.