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MODELLING AND SIMULATION APPLICATION BY THE OPTIMIZATION OF DEEP DRAWING PROCESS

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ABSTRACT

For the past ten years the modelling and simulation is most often applied in manufacturing *activities for the purpose of optimization, controlling and increasing of the productivity of manufacturing processes. Mathematical modelling and simulation are superstructure methods whose basic aim is the innovation of existing processes, modernizing and elevation on higher technological-economic level. Defined model is foundation of analysis and simulation of process which needs to enable the choice of a optimal solution.*

By means of modelling of the deep drawing process of axis – symmetric workpieces has been defined a mathematical model of drawing force process $F_d = f(r_M, \mu, \varphi)$ *, that is the optimal range of the roundness radius of the die* r_M *, the optimal range of degree of strain* φ *and the optimal of* tehnological lubricator μ. The mathematical models, which shows dependency of deep drawing *force and the parameters of drawing process, the geometry of tool and the tribological conditions of the plastic forming, were estimated by regression analysis:* $F_d = f(\varphi)$, $F_d = f(r_M)$ i $F_d = f(\mu)$. The results of performed research show that the *mathematical – experimental modelling can be successfully used for defining the form of the tool work zone, the parameters of tribological processes and the technological parameters of deep drawing process.*

1. INTRODUCTION

Modelling and simulation of machining processes needs to realize a efficacious and effective manufacturing which means of elimination or advancement of a passe′ processes with the reducing of a manufacturing expenses and shortening a deadlines of delivery in order to achieve a competitive advantage.

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An optimal choice of the machining procedure influences directly on:

- the minimization of machining time per product unit,
- enlargement of process productivity and usability of machining system,
- increasing product quality and
- reduction of preparation expenses and making.

Consequently, and machining technologies and processes, that have been applied for a number of years in definite conventional form can be innovated by applying corresponding modelling methods without considerable financial investment but by using high knowledge from the area of: modelling, simulations, optimizations, theory of processes, computer technique and other close areas (Figure 1.) [1,2,3,4,5,6,7,8,9,10].

Figure 1. Classical and modern machining process

An optimal technological forming process by means of plastic forming consists of its planning and execution within the optimal constructional (tools), tribological (lubricants used), technological (the strain, the temperature and the strain rate) and energetic (deforming force and work) conditions of forming. Due to very complicated work conditions and numerous influential parameters in the process of deep drawing of metal sheets, this process is one of the most complicated ones in the field of plastic forming.

Because of that the field of deep drawing attracts the attention of a large number of researchers $[1,4,5,9,10,11,12,13,14,15,16]$. The aim of this thesis is to research and model the influence of the parameters of tool construction (geometry of the deformation zone), the forming process (degree of strain) and tribology (lubrication) to the drawing force.

2. MATHEMATICAL MODELLING METHODS

Modelling methods have been improved by development of: applied mathematics, mathematical statistics, operational researches, experimental and information-computational methods. Today, there are more different modelling methods. The use of the existing methods depends on: objects of modelling, required degree of accuracy of model, process kind, available equipment, necessity of optimisation and building of controlling systems [2,3,4,6,7,8]. The mathematical modelling methods are illustrated in Figure 2.

Figure 2. Mathematical modelling methods

3. OPTIMIZATION METHODS

The basic optimization methods, which using by defining optimal solution at machining processes, are shown on Figure 3. [2,3,6,7,8,16].

Figure 3. Optimization methods

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4. MODELLING OF DEEP DRAWING PROCESS

Machining processes have a stochastical character as other processes in technics so it is often used empirical-statistical modelling which gives the most accurate results in relation to other methods of modelling. A fundamental mathematical starting point for investigation and analysis of stochastical forming processes is the theory of probability with stochastical processes theory. Elaboration of stochastical model is based on statistical processing of experimental data [1,4,5,9]. Methods of defining of stochastical model can be based on processing of random experimental data, when conditions are not programed (passive experiment), and on data processing, when conditions of experiment are programed by application of mathematical theory of experimental design (active experiment), (Figure 4).

Figure 4. General scheme of elaboration algorithm of stochastical mathematical model

By application of active experiment it is possible to define the accurate mathematical model with minimal number of experimental data, which are achived by programmed variation of the input variables of process with pre-set limits of varying. Modelling and simulation of machining process

means defining the correlation among the parameters of the process in order to improve the existing processes and to raise them to a higher techno-economic level.

4.1. VARIABLES OF DEEP DRAWING PROCESS

The deep drawing process modelling has been performed for the three variables of process (Figure 5. & Figure 6.): geometry of tool (roundness radius of the die r_M), technological lubricanttribological process (contact friction μ) and technological process (degree of strain φ) [1].

Figure 5. Experimental tool for measurement of contact stresses in deep drawing process 1-punch, 2-blankholder, 3-workpiece, 4-die, 5-normal contact stress measuring sensor p_n, 6-tangential contact stress measuring sensor τ_c

Figure 6. Scheme for deep drawing force modelling

4.2. EXPERIMENTAL ANALYSIS OF DEEP DRAWING PROCESS

The identification of forming process is performed analysis on the basis of known theoretical data, experimental investigations and simulation of state on the contact surface between tool and workpiece [1,4,9,17,18,19,20,21,22,23,24,25,26,27].

4.2.1. Experimental sensor tools for contact stresses measurement

The measurement procedure and appropriate tools have been original developed for determination and analysis of contact stresses in deep drawing process [1,4,9,18,23,26]. The direct contact stresses determining method has been analyzed and on the basis of this, a sensor has been conceptually elaborated and designed for normal stresses $(p_n -$ sensor pin 1) and tangential contact stresses (τ_f – sensor pin 2) measurement (Figure 7). The contact stresses were measured by means of pins with tensiometer transducers, where force on the pin 1 is F_{nl} for $\theta = 90^\circ$ and on the pin 2 is *F₂* for θ < 90° or θ > 90°.

Figure 7. Experimental tool set-up

On the sensor pin 1: $F_{n1} = p_{n1} A$ On the sensor pin 2: $F_2 = p_2 A$ or $F_2 = F_x + F_f = p_x A + \tau_f A \tan \alpha$, that is pressure: $p_2 = p_x + \tau_f \tan \alpha$ or $p_x = p_2 - \tau_f \tan \alpha$, i.e. $p_{n1} = p_x$. The tangential contact stress (contact friction stress):

$$
\tau_f = \frac{p_2 - p_x}{\tan \alpha} \,. \tag{1}
$$

Coefficient of friction contact on die rounding:

$$
\mu = \frac{\tau_f}{p_{n1}} = \frac{\tau_f}{p_x} \text{ or } \mu = \frac{F_2 - F_{n1}}{A \tan \alpha \ p_{n1}}.
$$
 (2)

Contact friction forces: $F_{f1} = \tau_f A$ and $F_{f2} = \tau_f \frac{A}{\cos \alpha}$.

4.2.2. Experimental conditions and results

The experiment were performed on the metal sheet DIN St14, material thickness $s = 0.8$ mm and yield strength σ_f = 208-220 N/mm². Experiment were carried out by hydraulic press HSO-1-63 with following features: punch velocity $v = 10$ mm/s, maximal force 630kN, maximal stroke 250 mm. Blankholder pressure $p_d = 1.5 \text{ N/mm}^2$. Intensity of deep drawing force was carried out by the five different contact state between sheet and tool, that is: fragmentary dry friction (S), non phosphate sheet surface and deep drawing oil (U), phosphate surface and deep drawing oil (F+U), non phosphate sheet surface and molybdenum disulfide (MD), phosphate sheet surface and molybdenum disulfide (F+MD). Depends on lubricant, friction coefficient values has been determined by contact stresses measurement. The obtained results of friction coefficient depend on lubricant and degree of strain are presented in Table 1.

| Test | Input parameters | | Coded parameters | | Measured values of | |
|----------------|------------------|-----------|------------------|-----------|----------------------|--|
| number | λ^* | φ | X_I | X_2 | friction coefficient | |
| | $0,45$ (MD) | 0,40 | -1 | -1 | 0,040 | |
| $\overline{2}$ | $1,55$ (U) | 0,40 | | -1 | 0,115 | |
| 3 | $0,45$ (MD) | 0,60 | -1 | | 0,047 | |
| 4 | $1,55$ (U) | 0,60 | | | 0,124 | |
| 5 | $1,0$ (F+U) | 0,50 | θ | 0 | 0,082 | |
| 6 | $1,0$ (F+U) | 0,50 | θ | Ω | 0,081 | |
| | $1,0$ (F+U) | 0,50 | θ | Ω | 0,075 | |
| 8 | $1,0$ (F+U) | 0,50 | Ω | 0 | 0,083 | |
| 9 | $1,0$ (F+U) | 0,50 | Ω | Ω | 0,079 | |
| 10 | $0,22$ (F+MD) | 0,50 | $-1,4142$ | θ | 0,018 | |
| 11 | 1,78(S) | 0,50 | 1,4142 | 0 | 0,150 | |
| 12 | $1,0$ (F+U) | 0,35 | θ | $-1,4142$ | 0,074 | |
| 13 | $1,0$ (F+U) | 0,65 | θ | 1,4142 | 0,089 | |

Table 1. Experimental values of friction coefficient according to model (2)

* $\lambda_{MD} = \frac{\mu_{MD}}{\mu_{F+U}}$ 0,49(0,45) + $\lambda_{\rm MD} = \frac{\mu_{\rm MD}}{\mu_{\rm F+U}} = 0,49(0,45)$, $\lambda_{\rm U} = \frac{\mu_{\rm U}}{\mu_{\rm F+U}}$ 1,54(1,55) + $\lambda_{\mathrm{U}} = \frac{\mu_{\mathrm{U}}}{\mu_{\mathrm{F+U}}}$, $S = \frac{\mu_S}{\mu_{F+U}}$ 1,86(1,78) + $\lambda_S = \frac{\mu_S}{\mu_{F+U}} = 1,86(1,78)$, $\lambda_{F+MD} = \frac{\mu_{F+MD}}{\mu_{F+U}} = 0,22$

4.2.3. Sensor for deep drawing force measurement

Measurement of the friction coefficient were conducted by direct method with the aid of the strain gauges which were mounted on the die. Measurement of the deep drawing force were carried out by force transducer by Hottinger Baldwin Messtechnik (Figure 8.) [4].

Figure 8. Experimental tool for measurement of contact stresses and force in deep drawing

4.2.4. Experimental results for deep drawing force

The obtained results based on experimental research for deep drawing force F_i and coefficient of friction μ are shown in Table 2.

| Tests | | Input variables | | Experimental results | | |
|----------------------------------|--|--|--|--|--|--|
| N_j | used lubrication | r_M mm | φ | coefficient of friction | deep drawing force F_i daN | |
| $\mathbf{1}$ 234567 8 | MD \overline{U} MD U MD U MD U | 2,00 2,00 3,00 3,00 2,00 2,00 3,00 3,00 | 0,40 0,40 0,40 0,40 0,60 0,60 0,60 0,60 | 0,040(0,040) 0,115(0,120) 0,040(0,040) 0,115(0,120) 0,047(0,040) 0,124(0,120) 0,047(0,040) 0,124(0,120) | 962 1150 884 1031 1138 1293 1036 1195 | |
| 9 10 11 12 13 14 | $F+U$ $F+U$ $F+U$ $F+U$ $F+U$ $F+U$ | 2,50 2,50 2,50 2,50 2,50 2,50 | 0,50 0,50 0,50 0,50 0,50 0,50 | 0,082(0,080) 0,082(0,080) 0,075(0,080) 0,082(0,080) 0,080(0,080) 0,085(0,080) | 1148 1130 1115 1168 1114 1152 | |
| 15 16 17 18 19 20 | $F+MD$ S $F+U$ $F+U$ $F+U$ $F+U$ | 2,50 2,50 1,60 3,40 2,50 2,50 | 0,50 0,50 0,50 0,50 0,33 0,67 | 0,018(0,013) 0,150(0,147) 0,082(0,080) 0,082(0,080) 0,074(0,080) 0,089(0,080) | 956 1140 1203 1037 977 1228 | |

Table 2. Overview of experimental results: deep drawing force and coefficient of friction

Values above mentioned in bracket are values for five level of friction coefficient according to design of experiment. Those values are approximately exact to the measurement values (Table 2.)

4.3. DETERMINATION OF THE MATHEMATICAL MODEL

Drawing force is modelled by means of second rank polynomial for the three variables:

$$
Y = F_d = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_3 + b_{11} X_1^2 + b_{22} X_2^2 + b_{33} X_3^2 + b_{12} X_1 X_2 + b_{13} X_1 X_3 + b_{23} X_2 X_3 + b_{12} X_1 X_2 X_3
$$
\n(3)

On the basic of the calculated coefficients the obtained mathematical model is the following:

$$
Y = F_d = 1134,266 + 75,187X_1 - 49,99X_2 + 77,65X_3 - 34,61X_1^2 - 11,71X_2^2 - 20,52X_3^2 - 7,5X_1X_2 + 5X_1X_3 + 2,5X_2X_3 + 2,5X_1X_2X_3
$$
\n
$$
(4)
$$

Considering the significant repression coefficient, the mathematical model of drawing force is the following:

$$
F_d = -98.6 + 5340.67\mu - 21631.25\mu^2 + 134.22r_M - 46.84r_M^2 + 2828.5\varphi - 2052\varphi^2 \tag{5}
$$

Since $F_a = 0.251 \le F_t (5.7) = 3.97$, the mathematical model describes well the deep drawing process force, which depends on tool geometry, coefficient of contact friction, and degree of strain. Multiple regression coefficient $R = 0.911$ shows a good interdependency of the variables X_i and $Y_i = F_d$.

5. SIMULATION AND OPTIMIZATION OF DEEP DRAWING PROCESS

The function with the aim $F_c = F_d = F_d(X_l, X, Y_s)$ for the range $-1,682 \le X_i \le +1,682$ has the maximum $F_c = F_d_{max}$ for coded values $X_1 = X_{10}$, $X_2 = X_{20}$, $X_3 = X_{30}$, or for physical values $\mu = \mu_0$, r_M $=r_{M0}$ and $\varphi = \varphi_0$. We get the maximum force of drawing for the values: $\mu = 0.139$; $r_M = 1.30$ mm and φ = 0,693.

By orthogonal sections of the function with the aim $F_c = F_d$ through the point of maximum drawing force $F_{d \text{max}}(X_{10}, X_{20}, X_{30})$, we can obtain mathematical models of drawing force, where the force of the drawing process is determined by one variable:

$$
F_{\mu} = 927,39 + 5743,5\mu - 21631,25\mu^2
$$
 (6)

$$
F_{r_M} = 1210,56 + 135,8r_M - 46,84r_M^2
$$
\n(7)

$$
F_{\varphi} = 385,15 + 2753,5\varphi - 2052\varphi^2 \tag{8}
$$

According to the performed research and analysis the optimal range of deep drawing process (Figure 9.), for the given conditions of the experiment, is determined by the following values: $0.02 \leq \mu \leq 0.08$; $2.00 \leq r_M \leq 3.0$; $0.30 \leq \varphi \leq 0.50$.

The total optimal range of the deep drawing process for the first operation and the presented technological, constructional and tribological parameters is determined by the following values of the process: $0.04 \le \mu \le 0.08$; $2.00 \le r_M \le 2.50$; $0.40 \le \varphi \le 0.50$.

Increasing of deep drawing force depends on coefficient of friction μ and die radius r_M for constant values of strain degree $\varphi = 0.5$; from $F_{min} = 797,821$ daN for $r_M = 3,4$ mm and $\mu = 0.013$, up to F_{max} = 1228,24 daN for r_M = 1,6 mm and μ = 0,119 is shown in Figure 10 [4].

Figure 9. Drawing force curves and the optimal range of the deep drawing process roundness radius r_M and friction coefficient μ and b. degree of strain φ

On Figure 11 is shown deep drawing force depends on die radius r_M and strain degree φ for constant values of coefficient of friction $\mu = 0.08$.

Figure 10. Forming force vs coefficient of friction (μ) and die radius (r_M)

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Figure 11. Forming force vs die radius (r_M) *and strain degree (* φ *)*

6. CONCLUSIONS

On the basis of the performed research and the obtained results, the following conclusions can be drawn:

- Modelling and simulation methods provides wider possibilities in the solving of the problems of a machining process. Needs for the exact presented of process are higher with the development of modern and intelligent machining systems, computational technique and information technologies in what the procedures of modelling and simulation have a especially meaning.
- The optimum range of the deep drawing process for the first operation and the presented technological, constructional and tribological parameters is determined by the following values of the deep drawing process: $0.02 \le \mu \le 0.08$; $1.75 \le r_M \le 3.0$; $0.30 \le \varphi \le 0.50$.
- By means of a special measuring instrument experimental research of mechanical load on contact surface of drawing and rolling tools was carried out (τ_h, p_n) . The experimentalmathematical method described shows that the normal contact pressure and specific frictional force (tangential contact stress) can exactly be determined on the contact surface of tool and workpiece, which is significant for the proper tool construction, less energy consumption and less production costs.
- The performed research shows that the modelling and simulation methods can be successfully used for defining the form of the work zone of the tools, tribological processes and the parameters of the forming process by means of deep drawing.

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PRIMJENA MODELIRANJA I SIMULACIJE U OPTIMIZACIJI PROCESA DUBOKOG IZVLAČENJA

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REZIME

U posljednjih deset godina modeliranje i simulacija se najviše primijenjuju u proizvodnim djelatnostima u svrhu optimizacije, upravljanja i povećanja produktivnosti obradnih procesa.

Matematičko modeliranje i optimizacija su metode nadgradnje čiji je osnovni cilj inoviranje postojećih procesa, moderniziranje i podizanje na viši tehnološki i ekonomski nivo. Definirani model je temelj analize i simulacije procesa koji treba omogućiti izbor optimalnog rješenja. Izbor optimalne tehnologije i/ili postupka obrade je najčešće osnovni utjecajni element proizvodnosti i ekonomičnosti sistema. Optimalnim izborom obradnog procesa izravno se utječe na minimizaciju vremena izrade po jedinici proizvoda.

Modeliranje i simulacije pružaju šire mogućnosti u rješavanju problematike obradnih procesa. Potrebe za egzaktnim predstavljanjem procesa sve su veće razvojem modernih i inteligentnih obradnih sistema, kompjutorske tehnike i informacijhskih tehnologija u čemu metode modeliranja i simulacije imaju posebno značenje.

*U radu je specijalno izrađenom mjerno-senzorskom aparaturom eksperimentalno ispitivano mehaničko opterećenje kontaktne površine (*τ*t, pn) alata za duboko izvlačenje. Prikazana eksperimentalna metoda pokazuje da se na kontaktnoj površini alata i obratka mogu odrediti kontaktni normalni pritisak i specifična sila trenja (tangencijalno kontaktno naprezanje), što je značajno za pravilnu konstrukciju alata, manju potrošnju energije i niže troškove izrade. Rješenje ovih problema usko je povezano sa istraživanjem triboloških procesa koji se dešavaju na kontaktnoj površini alata.*

Primjenom modeliranja procesa dubokog izvlačenja osnosimetričnih izradaka definiran je matematički model sile procesa izvlačenja ^{F_d = f(r_M,μ,φ)_, *odnosno optimalno područje radijusa zaobljenja matrice*}

alata ^IM , optimalno područje stupnja deformacije φ i optimalno tehnološko sredstvo za podmazivanje ^μ*. Matematički modeli koji opisuju ovisnost sile dubokog izvlačenja o parametrima procesa izvlačenja, geometriji alata i tribološkim uvjetima plastičnog oblikovanja određeni su relacijom:* $F_d = f(\varphi)$, $F_d = f(r_M)$ i $F_d = f(\mu)$ _.

Optimalni oblik geometrije alata za izvlačenje i povoljni kontaktni uvjeti smanjuju naprezanja i silu izvlačenja i povećavaju postojanost alata i tehno – ekonomske učinke obradnog procesa. Rezultati izvedenog istraživanja pokazuju da se metode modeliranja, simulacije i optimizacije mogu uspješno primijeniti za definiranje oblika radne zone alata, triboloških procesa i tehnoloških parametara procesa dubokog izvlačenja.