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THEORETICAL SOLUTION OF CONTACT STRESSES IN UPSETTING OF CYLINDER BY SPHERICAL CONCAVE DIES

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ABSTRACT

Determination of stress-strain state in the technology of plasticity is essential as the knowledge of this state enables the determination of basic process parameters such as: deformation force, mean pressure and work of deformation. Furthermore, it makes possible to analyze the potential of material formability. In this paper theoretical analysis of contact stresses in upsetting of cylinder by spherical concave dies has been presented.

1. INTRODUCTION

Technological methods of upsetting are the basic operations which take place in many bulk multi phase metal forming processes. Example for that are processes of forging and extrusion in which operations of upsetting are present at preliminary phase of forming. Furthermore, operations of upsetting can also be used for determination of yield stress and some parts of formability limit diagram.

Today, many different methods of upsetting are present in industrial production. Operation of upsetting of cylinder by spherical concave dies takes part in many industrial processes, such as: forming of a car's brace, forming of a bearing's ball, forming of a screw's head and forming of a spherical engraving (Fig.1).

Theoretical analysis of stress-strain state, presented in this paper, enables the determination of stress components on a contact surface between a workpiece and a die. Knowledge of contact stresses is enough condition for determination of basic process parameters (forming load and work of deformation) what is necessary for correct selection of machine and for constructing of dies.

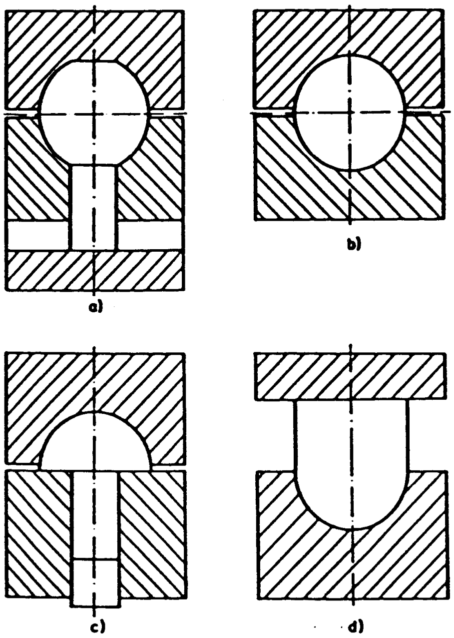

Figure 1. Examples for forming parts with spherical working surfaces

2. ANALYSIS OF STRESS-STRAIN STATE IN UPSETTING OF CYLINDER BY SPHERICAL CONCAVE DIES

The scheme of upsetting of cylinder by spherical concave dies with stress components is shown on Fig.2. Determination of stress components has been accomplished by solving approximate equilibrium equation with using of simplified yield criteria (so-called slab method). This procedure of analysis requires introduction of certain assumptions and simplifications what enables getting of concrete solutions:

- All plane sections parallel to the axis of cylinder remain plane during the upsetting;

- Tangential contact stress τ_k is result of friction on the contact surface of dies and billet and it is proportional to normal contact stress σ_n and to friction coefficient μ: $\tau_k = \mu \cdot \sigma_n$;

- Radial stress σ_r equals tangential stress σ_t in any section: $\sigma_r = \sigma_t$;

- Yield stress K is not constant (K≠cons.) in the deformation zone what agrees with the real process of upsetting.

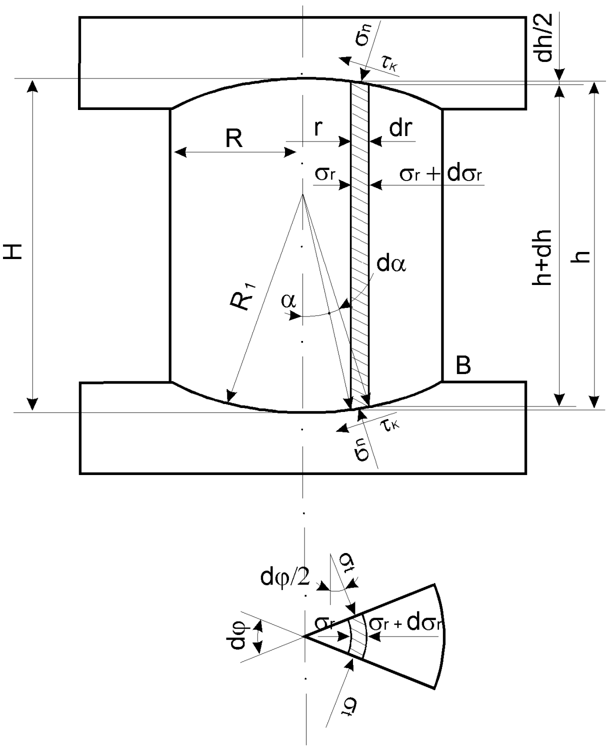

Figure 2. Stress scheme in upsetting of cylinder by spherical concave dies

In accordance with the stress scheme presented on Fig.2, basic equilibrium equation is:

$$\sigma_r \cdot r \cdot d\varphi \cdot h - (\sigma_r + d\sigma_r) \cdot (r + dr) \cdot (h + dh) \cdot d\varphi - 2 \cdot \sigma_n \cdot \frac{dr}{\cos\alpha} \cdot r \cdot d\varphi \cdot \sin\alpha -$$
$$- 2 \cdot \tau_k \cdot r \cdot d\varphi \cdot \frac{dr}{\cos\alpha} \cdot \cos\alpha + 2 \cdot \sigma_t \left(\frac{h + (h + dh)}{2} \right) \cdot dr \cdot \sin\frac{d\varphi}{2} = 0 \tag{1}$$

After transformations of equation (1) by using latter presumptions and simplifications, the following expression has been obtained:

$$\sigma_r \cdot \frac{dh}{dr} + \frac{d\sigma_r}{dr} \cdot h + 2 \cdot \sigma_n \cdot (tg\alpha + \mu) = 0 \tag{2}$$

Elimination of normal contact stress σ_n from equation (2) can be achieved by using following form of yield criteria:

$$\sigma_n = \sigma_r + K \tag{3}$$

So, equation (2) becomes:

$$\frac{d\sigma_r}{dr}\cdot h+\sigma_r\cdot\frac{dh}{dr}+2\cdot(\sigma_r+K)\cdot(tg\alpha+\mu)=0 \tag{4}$$

Geometrical relations who signify according to Fig.2 are:

$$r=R_1\cdot sin\,\alpha\,,\quad dr=R_1\cdot cos\,\alpha\cdot d\alpha$$

$$h=H-2\cdot R_1\cdot(1-cos\,\alpha)=2\cdot R_1\cdot\left(\frac{H}{2\cdot R_1}-1+cos\,\alpha\right)=2\cdot R_1\cdot(a+cos\,\alpha)$$

$$dh=-2\cdot R_1\cdot sin\,\alpha\cdot d\alpha$$

$$a=\frac{H}{2\cdot R_1}-1\,,\quad H=H_0-s$$

$$\frac{dh}{dr}=-2\cdot tg\alpha \tag{5}$$

After introduction of relations (5) into expression (4) the simplified form of equilibrium equation, expressed over the dependence of radial stress σ_r and contact angle α, has been obtained:

$$\frac{d\sigma_r}{d\alpha}+\frac{\mu\cdot\cos\alpha}{a+\cos\alpha}\cdot\sigma_r+K\cdot\frac{\sin\alpha+\mu\cdot\cos\alpha}{a+\cos\alpha}=0 \tag{6}$$

Yield stress K can be determined from the flow curve in following form:

$$K = K_0 + A\cdot\varphi_e^{\,b} \tag{7}$$

Constants K_0, A and b are experimental results for the given material.

Effective deformation φ_e is changeable through the section of billet during the upsetting of cylinder by spherical concave dies (Fig.3). Elementary zone with height h_0, on distance r_0 from the axis of cylinder (to whom responds contact angle α_0) moves, because of radial expansion, on distance r (with contact angle α) and compresses to height h because of construction of spherical dies. As a consequence, relation h_0/h changes in dependence of the billet’s radius and contact angle just as the effective deformation:

$$\varphi_e=ln\frac{h_0}{h}\Rightarrow\varphi_e=f(\alpha) \tag{8}$$

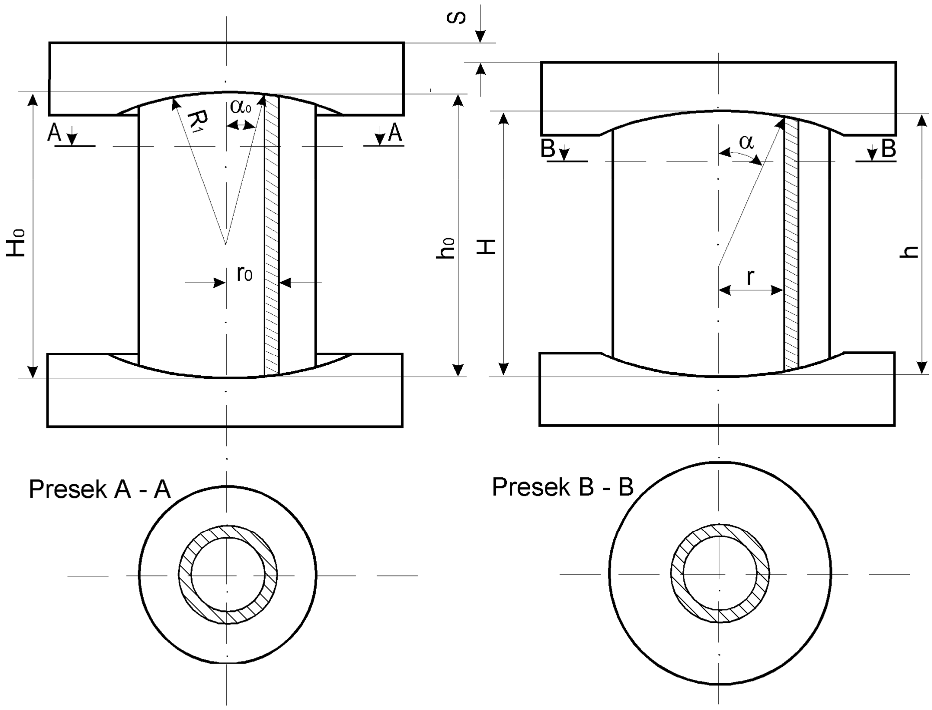

Figure 3. Behaving of cylindrical billet during the upsetting by spherical concave dies

Geometrical relations who follow from the Fig.3 are:

$h_0 = H_0 - 2R_1(1 - \cos\alpha_0)$ (9)

$h = H - 2R_1(1 - \cos\alpha)$ (10)

Where are: H_0 - initial billet's height for $\alpha=0$

H - billet's height for $\alpha=0$ after upsetting for the stroke s: $H = H_0 - s$;

h_0 - height of elementary zone before upsetting (with contact angle α_0);

h - height of elementary zone after upsetting for the stroke s (with contact angle α).

Height h can be determined only if we know the values of the contact angle α for each stroke and each elementary zone. Angle α is being determined by using the volume constant low:

V = cons.

$$r_0^{\,2} \cdot \pi \cdot h_0 + \frac{2}{3} \cdot \pi \cdot \left(r_0 \cdot tg \frac{\alpha_0}{2} \right)^2 \cdot \left(3R_1 - r_0 \cdot tg \frac{\alpha_0}{2} \right) =$$
$$= r^2 \cdot \pi \cdot h + \frac{2}{3} \cdot \pi \cdot \left(r \cdot tg \frac{\alpha}{2} \right)^2 \cdot \left(3R_1 - r \cdot tg \frac{\alpha}{2} \right) = const. \quad (10)$$

After substitution of expression (9) into equation (10) and use of geometrical relations: r = $R_1 \cdot \sin \alpha$ and r = $R_1 \cdot \sin \alpha$ we obtain:

$$\left(H_0 - 2R_1\right) \cdot cos^2 \alpha_0 + \frac{4}{3} \cdot R_1 \cdot cos^3 \alpha_0 - s = \left(H_0 - s - 2 \cdot R_1\right) \cdot cos^2 \alpha + \frac{4}{3} \cdot R_1 \cdot cos^3 \alpha \quad 11)$$

Equation (11) has been solved for different values of radius r_0: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 mm. As the result, pairs of points (α, φ_e) has been obtained. Then, these points has been approximated with the first-degree polynom and on this way the functional form of dependence between effective deformation and contact angle has been determined (Fig.4).

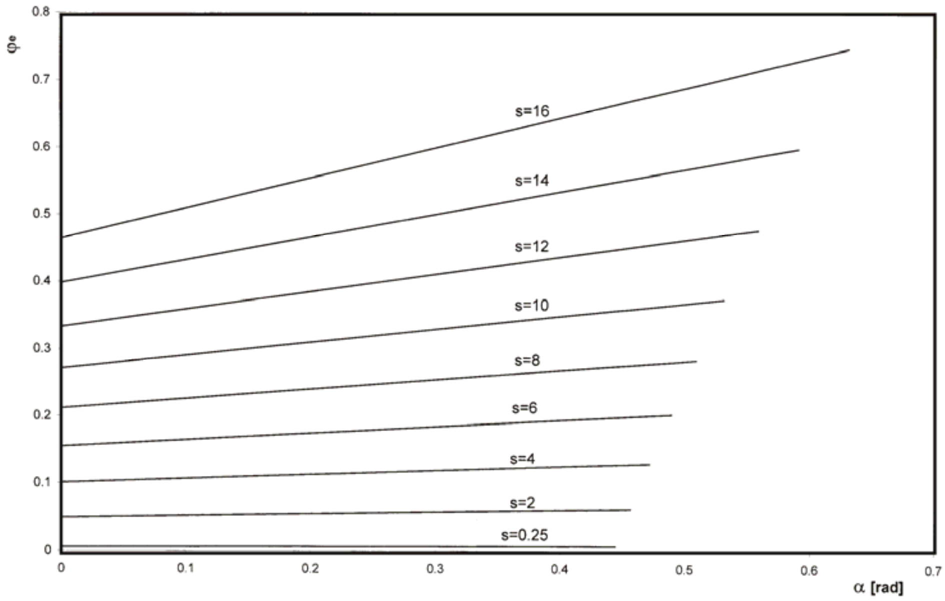

Figure 4. Dependence between effective deformation and contact angle

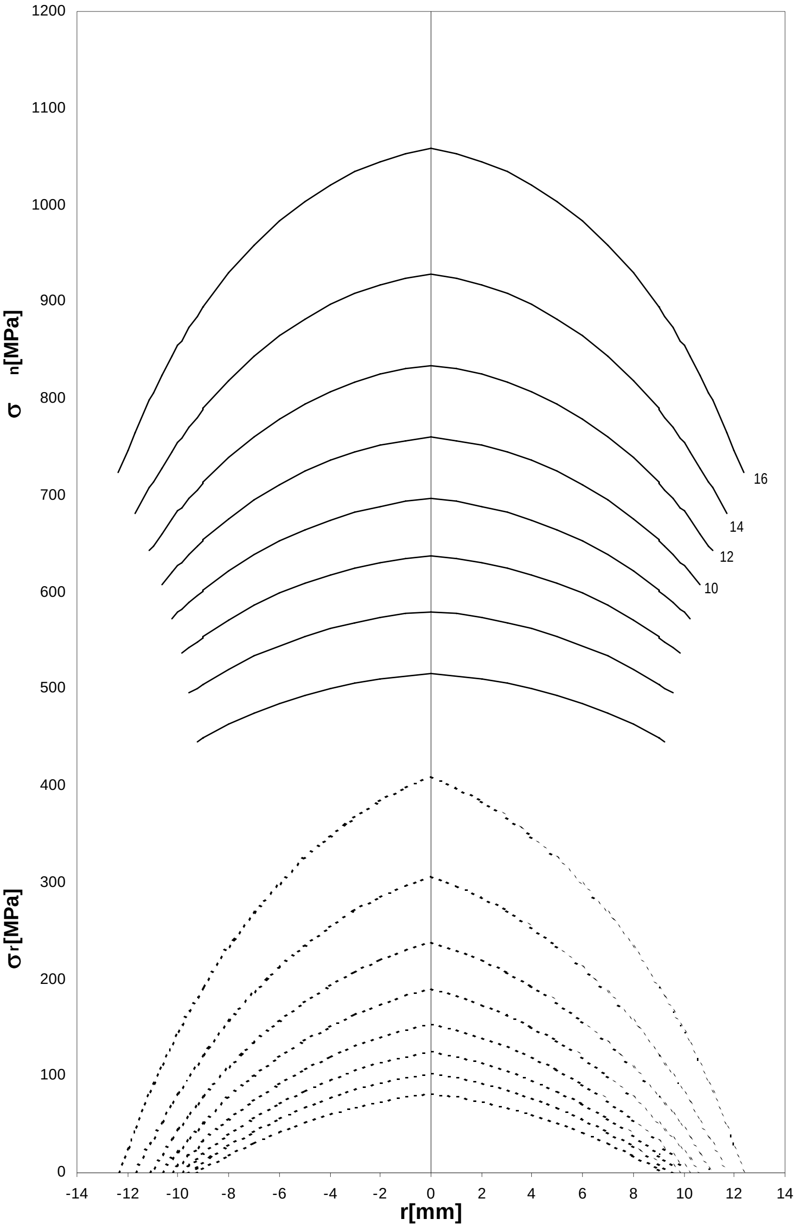

Figure 5. Distribution of normal and radial contact stress

Now, it was possible to obtain the final form of equilibrium equation expressed over the radial stress by substituting the expression (7) in equation (6):

$$\frac{d\sigma_r}{d\alpha} + \frac{\mu \cdot cos\,\alpha}{a + cos\,\alpha} \cdot \sigma_r + \left(K_0 + A \cdot \varphi_e{}^b\right) \cdot \frac{sin\,\alpha + \mu \cdot cos\,\alpha}{a + cos\,\alpha} = 0 \tag{12}$$

Elimination of radial contact stress σ_r from equation (12) has been achieved by using yield criteria and its differential in form:

$$\sigma_r = \sigma_n - K$$

$$\frac{d\sigma_r}{d\alpha} = \frac{d\sigma_n}{d\alpha} - \frac{dK}{d\alpha}$$

(13)

$$\frac{dK}{d\alpha} = \frac{d}{d\alpha}\left(K_0 + A \cdot \varphi_e{}^b\right) = A \cdot b \cdot \varphi_e{}^{b-1} \cdot \frac{d\varphi_e}{d\alpha} \tag{14}$$

According to this, the normal stress distribution over the die/billet interface surface is given by the following expression:

$$\frac{d\sigma_n}{d\alpha} + \frac{\mu \cdot cos\,\alpha}{a + cos\,\alpha} \cdot \sigma_n + \left(K_0 + A \cdot \varphi_e{}^b\right) \cdot \frac{sin\,\alpha}{a + cos\,\alpha} - A \cdot b \cdot \varphi_e{}^{b-1} \frac{d\varphi_e}{d\alpha} = 0 \tag{15}$$

Boundary condition that is necessary for solving the equation (12) and (15) is:

$$\alpha = \alpha_B = arcsin\frac{R}{R_1} \qquad \sigma_r = 0 \quad \text{tj. } \sigma_n = K$$

Solutions of equation (12) and (15) give normal stress σ_n and radial stress σ_r distribution along the section of billet (Fig.5). To solve these equations it is necessary to apply numerical integration. The results are interpolating functions that can be used for the further calculations.

For the illustration of shown analysis it has been used cylindrical specimen ϕ18x40mm made out of Č.1221 (DIN: CK-15).

Flow curve for the given material has been obtained experimentally by Rastegaev test and following expression was determined:

$$K = 325.455 + 438.233 \cdot \varphi_e^{0.471914} \quad [MPa]$$

3. CONCLUSION

Technological methods of upsetting have very important place in metal forming processes. Upsetting of cylinder by spherical concave dies represents the basic forming operation, which is present in different bulk metal forming processes. This operation found its place in industrial production in forming parts with different shapes and dimensions.

According to results shown on Fig.5 the following conclusions can be made.

The normal contact stress σ_n has positive sign during the whole process of upsetting what means that normal stress σ_n has pressure character along the whole billet section. Normal stress reaches its maximum on the axis of cylinder for values of die stroke. The values of the normal stress decreases toward the edge of the billet. With the increase of die stroke the values of normal stress also increases what can be explained by the increase of realized effective deformation.

Distribution of the radial stress σ_r is behaving very similar to the distribution of normal stress. Radial stress, as like as normal stress, has pressure character along the whole billet section. The values of the radial stress increases with the increase of die stroke and reach their maximum on the axis of cylinder. On the outer radius the values of the radial stress are always equal to zero for all die strokes.

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TEORIJSKO REŠENJE KONTAKTNIH NAPONA PRI SABIJANJU VALJKA SFERIČNIM KALUPIMA

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REZIME

Određivanje naponsko-deformacionog stanja u obradi deformisanjem veoma je važan zadatak jer omogućava određivanje osnovnih parametara procesa: deformacione sile, površinskog pritiska i deformacionog rada. Sem toga, omogućava i analizu deformabilnosti tj. obradivosti materijala.

U ovom radu prikazana je teorijska analiza kontaktnih napona za slučaj sabijanja valjka sferičnim kalupima. Određivanje naponskih komponenti izvršeno je rešavanjem približne diferencijalne jednačine ravnoteže uz primenu približne jednačine plastičnosti i odgovarajućih pretpostavki i uprošćenja. Pri tome je specifični deformacioni otpor smatran promenljivom veličinom. Rešenje diferencijalne jednačine ravnoteže dobijeno je numeričkim putem uz primenu odgovarajućih graničnih uslova.

Ilustracija dobijenih rešenja izvedena je na primerima sabijanja valjka polaznih dimenzija ϕ18x40mm od materijala Č.1221.