

# EFFECT OF STRESSES IN A THIN ROTATING DISK WITH EDGE LOAD FOR DIFFERENT MATERIALS

**Pankaj Thakur**

*Department of Mathematics, Indus International University Bathu,  
Una, Himachal Pradesh- 174301(INDIA)*

## ABSTRACT

*Elastic-plastic transitional stresses in a thin rotating disk with edge loading have been studied by using Seth's transition theory. Results have been discussed and presented graphically. It has been seen that higher value of angular speed is required for disc without edge loading. With the effect of edge loading, disc required lower angular speed. It has been seen that for incompressible material (i.e. rubber) higher angular speed is required for initial yielding as compare to disc made of compressible material (i.e. Lead, Copper and Steel). Rotating disk is likely to fracture by cleavage close to the inclusion at the bore.*

**Key words:** Rotating Disc, Load, Stresses, Transitional, material.

## 1. INTRODUCTION

Rotating disks are an essential part of the rotating machinery structure, e.g. rotors, turbines, compressors, flywheel and computer's disc drive. The analytical procedures presently available are restricted to problems with simplest configurations. The use of rotating disk in machinery and structural applications has generated considerable interest in many problems in domain of solid mechanics. Solutions for thin isotropic disks can be found in most of the standard elasticity and plasticity textbooks [1-5]. Guven [6] found the elastic-plastic stresses in a rotating annular disk of variable thickness and variable density under the assumptions of Tresca's yield condition, is associated with flow rule and linear strain hardening. To obtain the stress distribution, Guven matched the elastic-plastic stresses at the same radius  $r = z$  of the disc. Perfect elasticity and ideal plasticity are two extreme properties of the material and the use of ad-hoc like yield condition amount to divide the two extreme properties by a sharp line which is not physically possible. When a material passes from one state to another qualitatively different state, transition takes

place. Since this transition is non-linear in character and difficult to investigate, researchers have taken certain ad-hoc assumptions like yield condition, incompressibility condition and a strain law which may or may not be valid for the problem. Seth's transition theory [7] does not require these assumptions and thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out. This theory utilizes the concept of generalized strain measure and asymptotic solution at the critical points or the turning points of the differential equation defining the deformed field and has been successfully applied to a large number of the problems [7-14]. Seth [8] has defined the generalized principal strain measure as:

$$e_{ii} = \int_0^{e_{ii}^A} \left[ 1 - 2e_{ii}^A \right]^{\frac{n-1}{2}} d e_{ii}^A = \frac{1}{n} \left[ 1 - \left( 1 - 2e_{ii}^A \right)^{\frac{n}{2}} \right], \quad (i=1,2,3) \quad (1)$$

where 'n' is the measure and  $e_{ii}^A$  is the Almansi finite strain components [8]. For  $n = -2, -1, 0, 1, 2$  it gives Cauchy, Green Hencky, Swainger and Almansi measures respectively. In this paper, we investigate the problem of effect of stresses in a thin rotating disk with load edge for different materials by using Seth's transition theory. Results have been discussed and presented graphically.

## 2. MATHEMATICAL MODEL

We considered a thin annular disk of constant density with central bore of radius  $a$  and outer radius  $b$  as shown in Fig. 1. The disc, produced of material of constant density, is mounted on an edge loading. The disc is rotating with angular speed  $\omega$  about a central axis perpendicular to its plane. The thickness of disc is assumed to be constant and is taken sufficiently small so that the disc is effectively in a state of plane stress, that is, the axial stress  $T_{zz}$  is zero. The origin of the polar coordinate system  $r - \theta$  is assumed to be located at the centre of the disk and hole.

### Boundary Conditions

The disk considered in the present study is with variable Load. The inner surface of the disk is assumed to be fixed to a shaft. The outer surface of the disk is applied mechanical load. Thus, the boundary conditions of the problem are given by:

$$\begin{aligned} \text{(i)} \quad & T_{rr} = 0 \text{ at } r = a \\ \text{(ii)} \quad & T_{rr} = T_0 \text{ at } r = b \end{aligned} \quad (2)$$

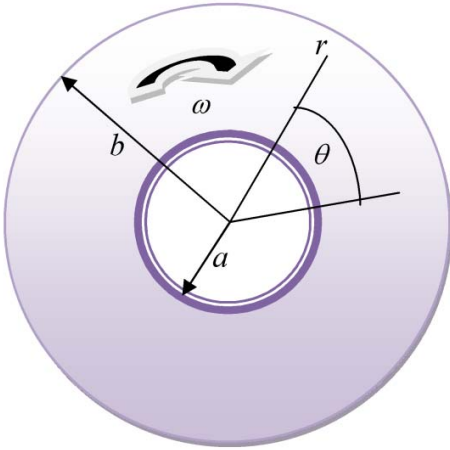
where  $T_{rr}$  and  $T_0$  denote stress along the radial direction and load.

### Formulation of the Problem

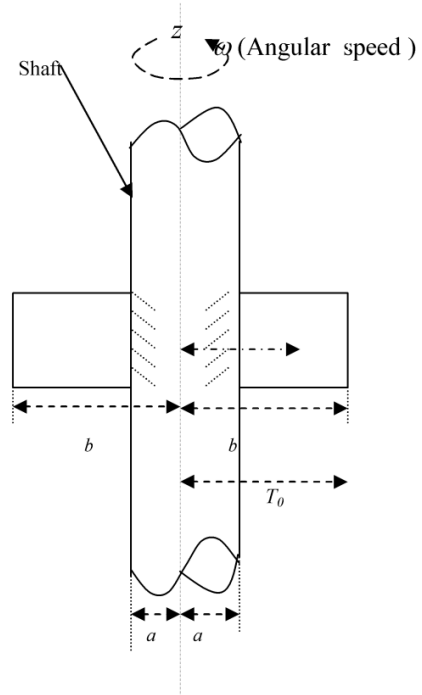
The displacement components in cylindrical polar co-ordinate are given by [8]:

$$u = r(1 - \beta), v = 0, w = dz \quad (3)$$

where  $\beta$  is position function, depending on  $r = \sqrt{x^2 + y^2}$  only, and  $d$  is a constant.



**Fig 1a** - Schematic diagram of a rotating disk with concentric circular hole



**Fig.1b** - Geometry of rotating disc

The finite strain components are given by Seth [8] as:

$$\begin{aligned} e_{rr}^A &= \frac{1}{2} [1 - (r\beta' + \beta)^2], & e_{\theta\theta}^A &= \frac{1}{2} [1 - \beta^2] \\ e_{zz}^A &= \frac{1}{2} [1 - (1-d)^2], & e_{r\theta}^A &= e_{\theta z}^A = e_{zr}^A = 0 \end{aligned} \quad (4)$$

where  $\beta' = d\beta/dr$  and meaning of superscripts "A" is Almansi.

By substituting eq. (4) in eq. (1), the generalized components of strain become:

$$\begin{aligned} e_{rr} &= \frac{1}{n} [1 - (r\beta' + \beta)^n], & e_{\theta\theta} &= \frac{1}{n} [1 - \beta^n], \\ e_{zz} &= \frac{1}{n} [1 - (1-d)^n], & e_{r\theta} &= e_{\theta z} = e_{zr} = 0 \end{aligned} \quad (5)$$

The stress-strain relations for isotropic material are given [5]:

$$T_{ij} = \lambda \delta_{ij} I_1 + 2\mu e_{ij}, \quad (i, j = 1, 2, 3) \quad (6)$$

where  $T_{ij}$  are stress components,  $\lambda$  and  $\mu$  are Lamé's constants,  $I_1 = e_{kk}$  is the first strain invariant,  $\delta_{ij}$  is the Kronecker's delta. Equation (6) for this problem becomes:

$$\begin{aligned} T_{rr} &= \frac{2\lambda\mu}{\lambda + 2\mu} [e_{rr} + e_{\theta\theta}] + 2\mu e_{rr} \\ T_{\theta\theta} &= \frac{2\lambda\mu}{\lambda + 2\mu} [e_{rr} + e_{\theta\theta}] + 2\mu e_{\theta\theta} \\ T_{r\theta} &= T_{\theta z} = T_{zr} = T_{zz} = 0. \end{aligned} \quad (7)$$

By substituting eq. (5) in eq. (7), the stresses are obtained as:

$$\begin{aligned} T_{rr} &= \frac{2\mu}{n} \left[ 3 - 2C - \beta^n \left\{ 1 - C + (2 - C)(P + 1)^n \right\} \right] \\ T_{\theta\theta} &= \frac{2\mu}{n} \left[ 3 - 2C - \beta^n \left\{ 2 - C + (1 - C)(P + 1)^n \right\} \right] \\ T_{r\theta} &= T_{\theta z} = T_{zr} = T_{zz} = 0 \end{aligned} \quad (8)$$

where  $C$  is the compressibility factor of the material in term of Lamé's constant, given by  $C = 2\mu / \lambda + 2\mu$ . The equations of motion are all satisfied except:

$$\frac{d}{dr} (rT_{rr}) - T_{\theta\theta} + \rho\omega^2 r^2 = 0 \quad (9)$$

where  $\rho$  is the density of the material of the rotating disc. By using eqs. (8) in eq. (9), one gets a non-linear differential equation for  $\beta$  as:

$$(2 - C)n\beta^{n+1}P(P + 1)^{n-1} \frac{dP}{d\beta} = \frac{n\rho\omega^2 r^2}{2\mu} + \beta^n \left\{ \begin{array}{l} 1 - (P + 1)^n \\ -nP \left[ \begin{array}{l} 1 - C \\ + (2 - C)(P + 1)^n \end{array} \right] \end{array} \right\} \quad (10)$$

where  $C$  is the compressibility factor of the material in term of Lamé's constant, given by  $C = 2\mu / \lambda + 2\mu$  and  $P$  is dependence function of  $\beta$  and  $\beta$  is dependence function of  $r$  only. From eq. (9), the turning points of  $\beta$  are  $P \rightarrow -1$  and  $\pm\infty$ .

### Solution through the principal stresses

For finding the plastic stress, the transition function is taken through the principal stress (see Seth [7, (see Seth [7,8], Gupta and Pankaj [9, 10, 11], Pankaj Thakur [12- 23]) at the transition point  $\tau \rightarrow \infty, \pm 1$ . The transition function  $\tau$  is defined as:

$$\tau = \frac{nT_{\theta\theta}}{2\mu} = 3 - 2C - \beta^n \left[ \begin{array}{c} 2-C \\ + (1-C)(P+1)^n \end{array} \right] \quad (11)$$

where  $\tau$  is function of  $r$  only and  $\tau$  is dimension.

Taking the logarithmic of eq. (11) with respect to  $r$ , one gets:

$$\log \tau = \log \left\{ 3 - 2C - \beta^n \left[ \begin{array}{c} 2-C \\ + (1-C)(P+1)^n \end{array} \right] \right\}$$

Differentiating with respect to  $r$ , one gets:

$$\therefore \frac{d}{dr}(\log \tau) = - \frac{1}{\left\{ \begin{array}{c} 3-2C \\ -\beta^n \left[ \begin{array}{c} 2-C \\ + (1-C) \\ (P+1)^n \end{array} \right] \end{array} \right\}} \cdot \frac{d}{dr} \left[ \begin{array}{c} 3-2C \\ -\beta^n \left[ \begin{array}{c} 2-C \\ + (1-C) \\ (P+1)^n \end{array} \right] \end{array} \right]$$

$$\therefore \frac{d}{dr}(\log \tau) = - \frac{\left[ \begin{array}{c} -n\beta^{n-1} \frac{d\beta}{dr} \left\{ 2-C + (1-C)(P+1)^n \right\} - \\ \beta^n \left\{ n(1-C)(P+1)^{n-1} \cdot \frac{dP}{d\beta} \cdot \frac{d\beta}{dr} \right\} \end{array} \right]}{\left\{ \begin{array}{c} 3-2C \\ -\beta^n \left[ \begin{array}{c} 2-C \\ + (1-C) \\ (P+1)^n \end{array} \right] \end{array} \right\}}$$

$$\therefore \frac{d}{dr}(\log \tau) = - \frac{\left[ \begin{array}{c} -n\beta^{n-1} \beta' \left\{ 2-C + (1-C)(P+1)^n \right\} - \\ \beta^n \left\{ n(1-C)(P+1)^{n-1} \cdot \frac{dP}{d\beta} \cdot \beta' \right\} \end{array} \right]}{\left\{ \begin{array}{c} 3-2C \\ -\beta^n \left[ \begin{array}{c} 2-C \\ + (1-C) \\ (P+1)^n \end{array} \right] \end{array} \right\}}$$

where  $P$  is function of  $\beta$ ,  $\beta$  is function of  $r$  and  $r\beta' = \beta P$ . Then

$$\therefore \frac{d}{dr}(\log \tau) = \left( - \frac{n\beta^n P}{r} \right) \frac{\left[ \begin{array}{c} 2-C + (1-C)(P+1)^{n-1} \\ \left\{ (P+1) + \beta \frac{dP}{d\beta} \right\} \end{array} \right]}{\left\{ 3-2C - \beta^n \left[ \begin{array}{c} 2-C + (1-C) \\ (P+1)^n \end{array} \right] \right\}} \quad (12)$$

By substituting the value of  $dP/d\beta$  from eq. (10) into eq. (12) and by taking asymptotic value  $P \rightarrow \pm\infty$ , one gets after integration:

$$R = K_1 r^{-1/(2-C)} \quad (13)$$

where  $K_1$  is a constant of integration, which can be determined by boundary condition and by  $\nu = (1-C)/(2-C)$  is the Poisson's ratio. From eq. (11) and (13), it follows:

$$T_{\theta\theta} = \left(\frac{2\mu}{n}\right) K_1 r^{-1/(2-C)} \quad (14)$$

By substituting eq. (14) into eq. (8), one gets:

$$rT_{rr} = \left[ K_2 + \frac{2\mu(2-C)K_1 r^{\frac{1-C}{2-C}}}{n(1-C)} \right] - \omega^2 \int r^2 \rho dr \quad (15)$$

where  $K_2$  is a constant of integration, which can be determined by boundary condition. By applying boundary condition (2) in eq. (15), one gets:

$$K_2 = -a^{\frac{1-C}{2-C}} \left[ \frac{bT_0 + \omega^2 \int_a^b r^2 dr}{\frac{1-C}{b^{2-C}} - a^{\frac{1-C}{2-C}}} \right] + \omega^2 \left[ \int r^2 \rho dr \right]_{at r=a}$$

$$K_1 = \frac{n(1-C)}{2(2-C)\mu} \left[ \frac{bT_0 + \omega^2 \int_a^b r^2 dr}{\frac{1-C}{b^{2-C}} - a^{\frac{1-C}{2-C}}} \right]$$

By substituting the value of  $K_1$  and  $K_2$  into equation (15), one gets:

$$T_{rr} = \left[ \frac{\frac{1-C}{r^{2-C}} - \frac{1-C}{a^{2-C}}}{r \left( \frac{1-C}{b^{2-C}} - a^{\frac{1-C}{2-C}} \right)} \left[ bT_0 + \frac{\rho\omega^2}{3} \left[ b^3 - a^3 \right] \right] + \frac{\rho\omega^2}{3r} \left[ \frac{a^3}{-r^3} \right] \right] \quad (16)$$

$$T_{\theta\theta} = \frac{r^{-\frac{1}{2-C}}(1-C) \left( bT_0 + \frac{\rho\omega^2}{3} [b^3 - a^3] \right)}{(2-C) \left( b^{\frac{1-C}{2-C}} - a^{\frac{1-C}{2-C}} \right)} \tag{17}$$

It is seen from eq. (14) that  $T_{\theta\theta}$  is maximum at the internal surface, therefore, yielding will take place at the internal surface and eq. (14) becomes:

$$|T_{\theta\theta}|_{r=a} = \left| \frac{a^{-\frac{1}{2-C}}(1-C)}{(2-C)} \left[ \frac{bT_0 + \frac{\rho\omega_i^2}{3} [b^3 - a^3]}{b^{\frac{1-C}{2-C}} - a^{\frac{1-C}{2-C}}} \right] \right| \equiv Y(say)$$

and angular speed  $\omega_i$  necessary for initial yielding is given by:

$$\Omega_i^2 = \frac{\rho\omega_i^2 b^2}{Y} = \frac{3b^2 \left( b^{\frac{1-C}{2-C}} - a^{\frac{1-C}{2-C}} \right) (2-C)}{(b^3 - a^3) a^{-\frac{1}{2-C}} (1-C)} - b\sigma_0 \tag{18}$$

where  $\sigma_0 = T_0 / Y$  and  $\omega_i = \frac{1}{b} \Omega_i (Y / \rho)^{1/2}$ .

We introduce the following non-dimensional components as:

$$R = r / b, R_0 = a / b, \sigma_r = T_{rr} / Y,$$

$$\sigma_\theta = T_{\theta\theta} / Y,$$

$$\Omega^2 = \rho\omega^2 b^2 / Y \text{ and } \sigma_0 = T_0 / Y.$$

Eqs. (16), (17) and (18) becomes:

$$\sigma_r = \left[ \left( \frac{R^{\frac{1-C}{2-C}} - R_0^{\frac{1-C}{2-C}}}{R \left( 1 - R_0^{\frac{1-C}{2-C}} \right)} \right) \left[ \sigma_0 + \frac{\Omega_i^2}{3} (1 - R_0^3) \right] - \frac{\Omega_i^2}{3R} (R^3 - R_0^3) \right] \tag{19}$$

$$\sigma_{\theta} = \frac{(1-C)R^{-\frac{1}{2-C}}}{(2-C)\left(1-R_0^{\frac{1-C}{2-C}}\right)} \left[ \sigma_0 + \frac{\Omega_i^2}{3}(1-R_0^3) \right] \quad (20)$$

$$\Omega_i^2 = \frac{3}{(1-R_0^3)} \left[ \frac{(2-C)\left(1-R_0^{\frac{1-C}{2-C}}\right)}{(1-C)R^{-\frac{1}{2-C}}} - \sigma_0 \right] \quad (21)$$

Eqs. (19) (20) and (21) give elastic-plastic transitional stresses and angular speed for thin rotating disc with loading edge. Stresses and angular speed give by eqn. (19) (20) and (21) for fully plasticity  $C = 0$  become:

$$\sigma_r = \left[ \left( \frac{\frac{1}{R^2} - R_0^{\frac{1}{2}}}{R(1-R_0^{1/2})} \right) \left[ \sigma_0 + \frac{\Omega_f^2}{3}(1-R_0^3) \right] - \frac{\Omega_f^2}{3R}(R^3 - R_0^3) \right] \quad (22)$$

$$\sigma_{\theta} = \frac{R^{-\frac{1}{2}}}{2\left(1-R_0^{\frac{1}{2}}\right)} \left[ \sigma_0 + \frac{\Omega_f^2}{3}(1-R_0^3) \right] \quad (23)$$

From eqn. (19) the angular speed  $\omega_f > \omega_i$  for which the disc becomes fully plastic  $C = 0$  at  $r = b$  is given by:

$$\Omega_f^2 = \frac{\rho\omega_f^2 b^2}{Y} = \frac{3}{(1-R_0^3)} \left[ 2(1-\sqrt{R_0}) - \sigma_0 \right] \quad (24)$$

where  $\omega_f = \frac{1}{b} \Omega_f (Y/\rho)^{\frac{1}{2}}$ .

### 3. RESULTS AND DISCUSSION

Curves have been drawn in Fig. 2, between angular speed required for initial yielding along the radius of a disc with and without edge loading. It has been seen that higher value angular speed is required for disc without edge loading.



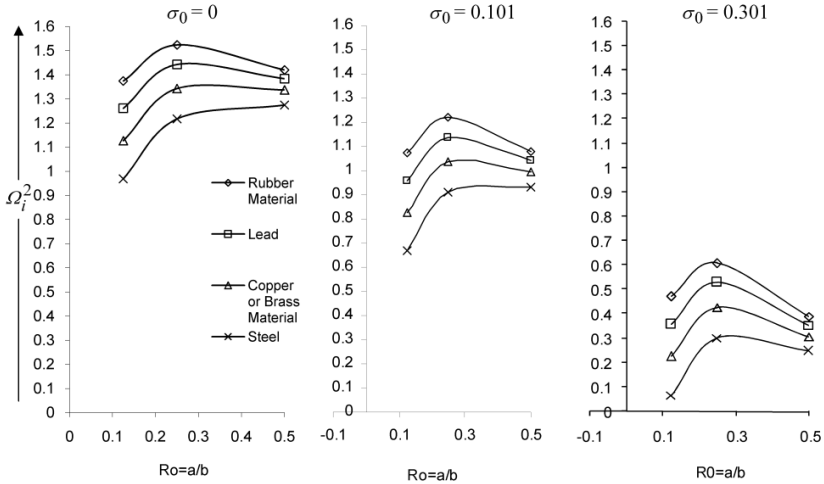
With the effect of edge loading, disc required lower angular speed. For incompressible material *i.e. rubber* higher angular speed is required for initial yielding as compare to disc made of Lead, copper or Brass and Steel materials.

**Table 1:** Angular speed required for initial yielding and fully plastic state.

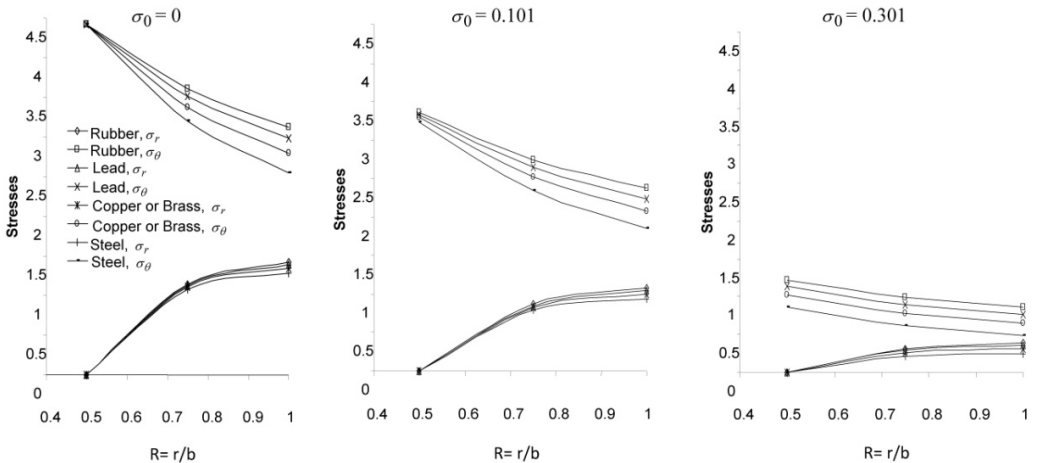
Load	Compressibility of material C	Angular speed required for initial yielding $\Omega_i^2$	Angular speed required for fully-plastic state $\Omega_f^2$	Percentage increase in angular speed $\left(\sqrt{\Omega_f^2 / \Omega_i^2} - 1\right) \times 100$
0.00	0 (Rubber material)	1.420161	2.008411	18.92071307 %
	0.25 (Lead material)	1.383601	3.008411	47.45623823 %
	0.5 (Copper or Brass material)	1.336737	4.008411	73.16620448 %
0.101	0 (Rubber material)	1.007304	5.008411	122.9819459 %
	0.25 (Lead material)	1.040744	6.008411	140.2745975 %
	0.5 (Copper or Brass material)	0.904092	7.008411	178.4219483 %
0.301	0 (Rubber material)	0.391589	8.008411	352.2285065 %
	0.25 (Lead material)	0.355029	9.008411	403.7234643 %
	0.5 (Copper or Brass material)	0.308165	10.008411	469.8898339 %

It can be also seen from Table 1 that for compressible material (*i.e. Lead, copper or Brass, Steel materials*) higher percentage increased in angular speed is required to become fully plastic as compared to rotating disc made of incompressible material (*i.e. rubber*). With the effect of loading edge, the percentage in angular speed much increased as compare to without edge loading.

Curves have been drawn in figure3, stresses distribution at elastic-plastic transitional state and fully plastic state of a disc with edge loading and radius  $R = r/b$ . It has been seen that the circumferential stresses has maximum value at the internal surface of the rotating disc made of rubber material as compare to Lead, Copper or brass and Steel materials. With the effect of edge loading stresses must be decreased with increase values of edge load.



**Fig.2 - Angular speed required for initial yielding along the radius of a disc with and without edge loading.**



**Fig. 3 - Stresses distribution at elastic-plastic transitional state and fully plastic state of a disc with edge loading and radius  $R = r/b$ .**

## 4. CONCLUSION

It has been seen that higher value of angular speed is required for disc without edge loading. With the effect of edge loading disc required lower angular speed. It has been seen that for incompressible material *i.e.* rubber required higher angular speed for initial yielding as compare to disc made of Lead, Copper or Brass and Steel materials. Rotating disk is likely to fracture by cleavage close to the inclusion at the bore.

---

---

**REFERENCES**

- [1] Timoshenko, S.P. & Goodier, J.N., *Theory of Elasticity*, McGraw-Hill, New York, 1970.
- [2] Blazynski, T.N., *Applied Elasto-plasticity of Solids*, McMillan Press Ltd., London, 1983.
- [3] Johnson, W. & Mellor, P.B., *plasticity for Mechanical Engineers*, Van-Nostrand Reinhold Company, London, 1962.
- [4] Chakrabarty, J., *Theory of Plasticity*, McGraw-Hill, New York, 1987.
- [5] Sokolnikoff I.S., *Mathematical theory of Elasticity*, 2<sup>nd</sup> edition, New York, pp. 65-79, 1952.
- [6] U. Guven, Elastic-plastic stresses in a rotating annular disc of variable thickness and variable density, *Archive of Appl. Mech.*, 61, 1991, pp. 548-552.
- [7] B. R. Seth, Transition theory of Elastic- plastic deformation, Creep and relaxation, *Nature*, 195, 1962, pp. 896-897.
- [8] B. R. Seth, Measure concept in Mechanics, *Int. J. Non-linear Mech.*, Vol. 1, No. 1, 1966, pp. 35- 40.
- [9] S.K. Gupta, Pankaj Thakur, Creep Transition in a thin rotating disc with rigid Inclusion, *Defence Science Journal*, India, Vol. 57 ,No. 2, 2007, pp. 185-195.
- [10] Pankaj Thakur, S. K. Gupta, Creep Transition in an isotropic disc having variable thickness subjected to internal pressure, *Proceeding National Academy of Science, India, Section- A* 78(I), pp. 57-66, 2008.
- [11] Pankaj Thakur, S. K. Gupta, Thermo Elastic-plastic transition in a thin rotating Disc with inclusion, *Thermal Science*, Vol. 11, No. 1, 2007, pp. 103-118.
- [12] Pankaj Thakur, Elastic - plastic transition stresses in a transversely isotropic thick-walled cylinder subjected to internal pressure and steady state temperature, *Thermal Science*, Vol. 13, No. 4, 2009, pp. 107-118.
- [13] Pankaj Thakur, Elastic-plastic transition stresses in a thin rotating disc with rigid inclusion by infinitesimal deformation under steady state Temperature, *Thermal Science*, Vol. 14, No. 1, 2010, pp. 209-219.
- [14] Pankaj Thakur, Creep Transition Stresses in a thin rotating disc with shaft by finite deformation under steady state temperature, *Thermal Science*, Vol. 14, No. 2, 2010, pp. 425-436.
- [15] Pankaj Thakur, Elastic - Plastic Transition Stresses In Rotating Cylinder By Finite Deformation Under Steady- State Temperature, *Thermal Science*, Vol. 15, No. 2, 2011, pp. 537-543.
- [16] Pankaj Thakur, Creep Transition stresses of a Thick isotropic spherical shell by finitesimal deformation under steady state of temperature and internal pressure, *Thermal Science International Scientific Journal*, Belgrade, Serbia and Montenegro, Vol. 15, Suppl. 2, pp. S157-S165, 2011.
- [17] Pankaj Thakur, Steady thermal stress and strain rates in a rotating circular cylinder under steady state temperature, *Thermal Science International Scientific Journal*, Vol. 17, No. 1or 2, 2013 DOI REFERENCE: 10.2298/TSCI110315080P.
- [18] Pankaj Thakur, Steady thermal stress and strain rates in a circular cylinder with non-homogeneous compressibility subjected to thermal load, *Thermal Science International Scientific Journal*, Vol. 17, No. 1or 2, 2013 DOI REFERENCE: 10.2298/TSCI110318079P.
- [19] Pankaj Thakur, Stresses in a thin rotating disc of variable thickness with rigid shaft, *Journal for Technology of Plasticity*, Serbia, Vol. 37, Number 1, pp. 1-14, 2012.

- [20] Pankaj Thakur, Elastic - Plastic Transition in a Thin Rotating Disc having variable density with Inclusion, *Structural Integrity and life, Serbia* , Vol. 9, No. 3, 2009, pp. 171-179.
- [21] Pankaj Thakur, Effect of transition stresses in a disc having variable thickness and Poisson's ratio subjected to internal pressure, *WSEAS TRANSACTIONS on APPLIED and THEORETICAL MECHANICS*, Vol. 6, No. 4, 2011, pp. 147-159.
- [22] Pankaj Thakur, Creep transition stresses in a spherical shell under internal pressure by using lebesgue measure concept, *International journal Applied Mechanics and Engineering*, Poland, Vol. 16, No. 3, 2011, pp. 83-87.
- [23] Pankaj Thakur, Deformation in a thin rotating disc having variable thickness and edge load with inclusion at the elastic-plastic transitional stresses *journal Structural Integrity and life, Serbia*, Vol.12, No1, 2012, pp.65-70 .

# NAPONSKO STANJE U TANKOM DISKU SA IVIČNOM SILOM ZA RAZLIČITE MATERIJALE DISKA

**Pankaj THAKUR**

*Department of Mathematics, Indus International University Bathu,  
Una, Himachal Pradesh- 174301(INDIA)*

## REZIME

*Rotirajući diskovi su jedan od osnovnih delova rotacionih mašinskih sturktura, kao što su rotori, turbine, kompresori, kompjuterski diskovi... Ranije izvedene analitičke procedure su ograničene samo na problem jednostavne konstrukcije. U ovom radu vršena je analiza napona u tankom rotirajućem disku sa ivičnom silom Naponi su izvedeni su za elasto-plastično i potpuno plastično stanje pomoću Seth-ove tranzicione teorije. Dobijeni rezultati su diskutovani i grafički prezentovani. Zaključeno je da je veća ugaona brzina neophodna kod diskova bez ivičnih sila. Takođe se zaključeno da su za materijale poput gume neophodne veće ugaone brzine za plastično tečenje materijala nego za metalne materijale poput olova, bakra i čelika. Veća je i verovatnoća da će doći do loma roirajućeg diska na mestima bliži otvoru.*

**Ključne reči:** *naponi na rotirajućem disku, pomeranja, rotirajući disk, ugaona brzina*