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STRESSES IN A THIN ROTATING DISC OF VARIABLE THICKNESS WITH RIGID SHAFT

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ABSTRACT

Stresses for the elastic-plastic transition and fully plastic state have been derived for a thin rotating disc with rigid shaft having variable thickness by using Seth's Transition theory and results have been discussed and depicted graphically. It has been observed that in the absence of thickness, rotating disc with inclusion and made of compressible material e.g. Copper, Brass and Steel, yields at the internal surface at a lesser angular speed as compared to a rotating disc made of incompressible material e.g. rubber whereas it requires a higher percentage increase in angular speed to become fully plastic. With the effect of variation thickness, higher angular speed is required to yield at the internal surface. It has been observed that the radial stress is maximum at the internal surface. With the effect of variable thickness it increases the value of radial and circumferential stress at the internal surface for transitional state, whereas it can be seen that rotating disc having variable thickness increases the values of radial and circumferential stress at the internal surface for fully-plastic state.

Key words:* s*tresses, displacement, rotating disc, angular speed, Inclusion, thickness

1. INTRODUCTION

Rotating discs form an essential part of the design of rotating machinery, namely rotors, turbines, compressors, flywheel and computer's disc drive etc. The analysis of thin rotating discs made of isotropic material has been discussed extensively by Timoshenko and Goodier [1] in the elastic range and by Chakrabarty [2] and Hropertiesfor the plastic range.Their solutionfor the problem of fully plastic state does not involve the plane stress condition, that is to say, we can obtain the same stresses and angular velocity required by the disc to become fully plastic without using the plane stress condition (i.e. =0). Gupta and Shukla [4] obtained a different solution for the fully plastic state by using Seth's transition theory and plane stress condition. This theory [5] does not required

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any assumptions like an yield condition, incompressibility condition and thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out. It utilizes the concept of generalized strain measure and asymptotic solution at critical points or turning points of the differential equations defining the deformed field and has been successfully applied to a large number of problems [4,10-15, 17- 33].
Seth [6] has defined the generalized principal strain measure as,

$$e_{ii} = \int_0^{\overset{A}{e}_{ii}} \left[1 - 2\overset{A}{e}_{ii}\right]^{\frac{n}{2}-1} d\overset{A}{e}_{ii} = \frac{1}{n}\left[1 - \left(1 - 2\overset{A}{e}_{ii}\right)\right], \text{(i,j=1,2,3)} \tag{1}$$

where 'n' is the measure and .is the Almansi finite strain components. For n =-2, -1,0, 1, 2 it gives Cauchy, Green Hencky, Swainger and Almansi measures, respectively.
Here, Analysis of Elastic-Plastic Transition stresses in a thin Rotating Disc having variable thickness with rigid shaft is investigated by Seth's transition theory. The thickness of disc is assumed to vary along the radius in the form:

$$h = h_0 (r/b)^{-k} \tag{2}$$

Where h_0 is the thickness at $r = b$ and k is the thickness parameter. Result obtained have been numerically and depicted graphically.

2. GOVERNING EQUATIONS

We consider a thin disc of variable thickness with central bore of radius *a* and external radius *b*. The annular disc is mounted on a rigid shaft. The disc is rotating with angular speed of gradually increasing magnitude around axis perpendicular to its plane and passed through the center as shown in Figure 1. The thickness h of disc is assumed to be vary radially and is taken sufficiently small so that the disc is effectively in a state of plane stress, that is, the axial stress is zero.
The displacement components in cylindrical polar co- ordinate are given by [6].

$$u = r(1-\beta), \; v = 0, \; w = dz \tag{3}$$

whereβ is function of $r = \sqrt{x^2 + y^2}$ only and d is a constant.
The finite strain components are given by Seth [6] as:

$$\begin{aligned}
\overset{A}{e}_{rr} &\equiv \frac{\partial u}{\partial r} - \frac{1}{2}\left(\frac{\partial u}{\partial r}\right)^2 = \frac{1}{2}\left[1 - (r\beta' + \beta)^2\right] \\
\overset{A}{e}_{\theta\theta} &\equiv \frac{u}{r} - \frac{u^2}{2r^2} = \frac{1}{2}\left[1 - \beta^2\right] \\
\overset{A}{e}_{zz} &\equiv \frac{\partial w}{\partial z} - \frac{1}{2}\left(\frac{\partial w}{\partial z}\right)^2 = \frac{1}{2}\left[1 - (1-d)^2\right] \\
\overset{A}{e}_{r\theta} &= \overset{A}{e}_{\theta z} = \overset{A}{e}_{zr} = 0.
\end{aligned} \tag{4}$$

where $\beta' = d\beta/dr$

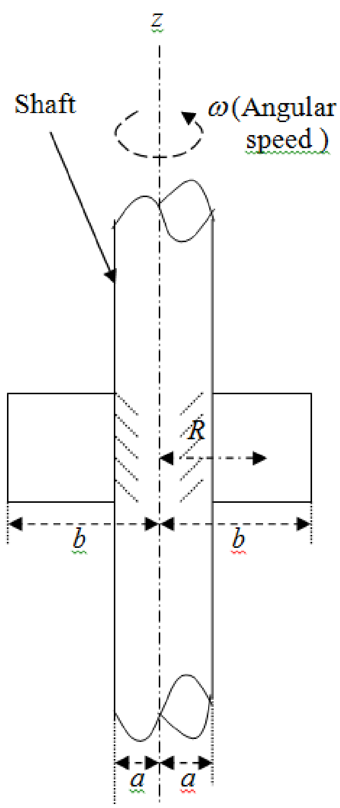

Fig. 1 - Geometry of Rotating Disc.

Substituting equation (4) in equation (1), the generalized components of strain are:

$$e_{rr} = \frac{1}{n}\left[1-(r\beta'+\beta)^n\right]$$

$$e_{\theta\theta} = \frac{1}{n}\left[1-\beta^n\right]$$

$$e_{zz} = \frac{1}{n}\left[1-(1-d)^n\right]$$

$$e_{r\theta} = e_{\theta z} = e_{zr} = 0 \qquad (5)$$

Where $\beta' = d\beta/dr$

The stress –strain relations for isotropic material are given by [16]

$$T_{ij} = \lambda\delta_{ij}I_1 + 2\mu e_{ij} \; (i,j=1,2,3) \qquad (6)$$

where T_{ij} and e_{ij} are the stresses and strain components, λ and μ are lame's constants and $I_1 = e_{kk}$ is the first strain invariant, δ_{ij} is the Kroncecker'sdelta.

Equations (6) for this problem become,

$$T_{rr} = \frac{2\lambda\mu}{\lambda+2\mu}\left[e_{rr}+e_{\theta\theta}\right]+2\mu e_{rr}$$

$$T_{\theta\theta} = \frac{2\lambda\mu}{\lambda+2\mu}\left[e_{rr}+e_{\theta\theta}\right]+2e_{\theta\theta}$$

$$T_{r\theta} = T_{\theta z} = T_{zr} = T_{zz} = 0 \qquad (7)$$

Using equation (4) in equation (6), the strain components in terms of stresses are obtained as [16]:

$$e_{rr} = \frac{\partial u}{\partial r} - \frac{1}{2}\left(\frac{\partial u}{\partial r}\right)^2 = \frac{1}{2}\left[1-(r\beta'+\beta)^2\right] = \frac{1}{E}\left[T_{rr} - \left(\frac{1-C}{2-C}\right)T_{\theta\theta}\right]$$

$$e_{\theta\theta} = \frac{u}{r} - \frac{u^2}{2r^2} = \frac{1}{2}\left[1-\beta^2\right] = \frac{1}{E}\left[T_{\theta\theta} - \left(\frac{1-C}{2-C}\right)T_{rr}\right]$$

$$e_{zz} = \frac{\partial w}{\partial z} - \frac{1}{2}\left(\frac{\partial w}{\partial z}\right)^2 = \frac{1}{2}\left[1-(1-d)^2\right] = -\frac{(1-C)}{E(2-C)}\left[T_{rr}-T_{\theta\theta}\right] \qquad (8)$$

And $e_{r\theta} = e_{\theta z} = e_{zr} = 0$

Where: $E = \dfrac{\mu(3\lambda + 2\mu)}{(\lambda + \mu)}$ and $C = \dfrac{2\mu}{\lambda + 2\mu}$

Substituting equation (5) in equation (7), we get the stress as:

$$T_{rr} = \frac{2\mu}{n}\left[3 - 2C - \beta^n\left\{1 - C + (2 - C)(P+1)^n\right\}\right]$$

$$T_{\theta\theta} = \frac{2\mu}{n}\left[3 - 2C - \beta^n\left\{2 - C + (1 - C)(P+1)^n\right\}\right]$$

$$T_{r\theta} = T_{\theta z} = T_{zr} = T_{zz} = 0 \tag{9}$$

Where $r\beta' = \beta P$.

Equations of equilibrium are all satisfied except

$$\frac{d}{dr}(rhT_{rr}) - hT_{\theta\theta} + \rho\omega^2 r^2 h = 0 \tag{10}$$

where ρ is the density of the material of the rotating disc.

Using equation (9) and (10), we get a non- linear differential equation in β as:

$$(2 - C)n\beta^{n+1}P(P+1)^{n-1}\frac{dP}{d\beta} = \left[\begin{array}{l}\dfrac{rh'}{h}\left[3 - 2C - \beta^n\left\{1 - C + (2 - C)(P+1)^n\right\}\right] + \dfrac{n\rho\omega^2 r^2}{2\mu} \\ + \beta^n\left[1 - (P+1)^n - np\left\{1 - C + (2 - C)(P+1)^n\right\}\right]\end{array}\right] \tag{11}$$

where $\beta' = d\beta/dr$ (P is function of β and β is function of r only).

From equation (11), the transition points of β are $P = -1$ and $\pm\infty$ .The boundary conditions are:

$u = 0$ at $r = a$ and $T_{rr} = 0$ at $r = b$ (12)

3. SOLUTION THROUGH THE PRINCIPAL STRESSES

It has been shown[4, 10-15, 17-33] that the asymptotic solution through the principal stress leads from elastic state to plastic state at transition point $P \to \pm\infty$, we define the transition function R as:

$$R = \frac{nT_{\theta\theta}}{2\mu} = \left[(3 - 2C) - \beta^n\left\{2 - C + (1 - C)(P+1)^n\right\}\right] \tag{13}$$

Taking the logarithmic differentiation of equation (13) with respect to r and using equation (11),we get:

$$\frac{d(\log R)}{dr} = -\frac{n\beta^n P}{r}\left[\frac{(2 - C) + (1 - C)(P+1)^{n-1}\left(P + 1 + \beta\dfrac{dP}{d\beta}\right)}{r\left[3 - 2C - \beta^n\left\{2 - C + (1 - C)(P+1)^n\right\}\right]}\right] \tag{14}$$

Taking the asymptotic value of equation (14) at $P \to \pm\infty$ and integrating, we get

$$R = \frac{A_1 r^{\nu-1}}{h} \tag{15}$$

where A_1 is a constant of integration.

From equation (13) and (15) and using equation (2),we have

$$T_{\theta\theta} = \left(\frac{2\mu}{n}\right)\frac{A_1 r^{\nu+k-1}}{h_0} \tag{16}$$

Substituting equation (16) in equation (10) using equation (2) and integrating, we get:

$$T_{rr} = \left(\frac{2\mu b^{-k}}{n\nu h_0}\right)A_1 r^{\nu-1} - \frac{\rho\omega^2 r^2}{(3-k)} + \frac{B_1 b^{-k}}{r h_0 r^{-k}} \tag{17}$$

whereB_1 is a constant of integration.

Substituting equations (16) and (17) in second equation of equation (8), we get

$$\beta = \sqrt{1 - \frac{2\nu}{E}\left[\frac{\rho\omega^2 r^2}{3-k} - \frac{B_1 b^{-k}}{h_0 r^{1-k}}\right]} \tag{18}$$

Substituting equation (18) in equation (3), we get

$$u = r - r\sqrt{1 - \frac{2\nu}{E}\left[\frac{\rho\omega^2 r^2}{3-k} - \frac{B_1 b^{-k}}{h_0 r^{1-k}}\right]} \tag{19}$$

where $E = \frac{2\mu(3-2C)}{(2-C)}$ is the Young's modulus.

Using boundary condition (12) in equations (16) and (17), we get

$$A_1 = \frac{\rho\omega^2 n\nu b^k h_0 (1-C)\left(b^{3-k} - a^{3-k}\right)}{2\mu(3-k)b^\nu} \tag{20}$$

$$B_1 = \frac{\rho\omega^2 a^{3-k} h_0}{(3-k)b^{-k}} \tag{21}$$

Substituting equations (20) and (21) in equations (16), (17), and (19) respectively, we get the transitional stresses and displacement as

$$T_{\theta\theta} = \frac{\rho\omega^2 \nu r^k \left(b^{3-k} - a^{3-k}\right)}{(3-k)r}\left(\frac{r}{b}\right)^\nu \tag{22}$$

$$T_{rr} = \frac{\rho\omega^2 r^k}{(3-k)r}\left[\left(b^{3-k} - a^{3-k}\right)\left(\frac{r}{b}\right)^\nu - r^{3-k} + a^{3-k}\right] \tag{23}$$

$$u = r - r\sqrt{1 - \frac{2\nu}{E}\frac{\rho\omega^2}{(3-k)r^{1-k}}\left[r^{3-k} - a^{3-k}\right]} \tag{24}$$

$$|T_{rr} - T_{\theta\theta}| = \left|\frac{\rho\omega^2 r^k}{(3-k)r}\left[(1-\nu)\left(b^{3-k} - a^{3-k}\right)\left(\frac{r}{b}\right)^{\nu} - r^{3-k} + a^{3-k}\right]\right| \tag{25}$$

From equation (25), it is seen that $|T_{rr} - T_{\theta\theta}|$ is maximum at the internal surface (that is at $r = a$), therefore yielding of the disc takes place at the internal surface of the disc and equation (25) can be written as:

$$|T_{rr} - T_{\theta\theta}|_{r=a} = \left|\frac{\rho\omega^2(1-\nu)\left(b^{3-k} - a^{3-k}\right)}{(3-k)a^{1-k}}\left(\frac{a}{b}\right)^{\nu}\right| = Y(say)$$

The angular speed necessary for initial yielding is given by:

$$\Omega_i^2 = \frac{\rho\omega_i^2 b^2}{Y} = \frac{(3-k)ab^2}{(1-\nu)a^k\left(b^{3-k} - a^{3-k}\right)}\left(\frac{b}{a}\right)^{\nu} \tag{26}$$

And $\omega_i = \frac{\Omega_i}{b}\sqrt{\frac{Y}{\rho}}$

The disc becomes fully plastic $(C \to 0)$ at the external surface and equation (25) becomes:

$$|T_{rr} - T_{\theta\theta}|_{r=a} = \left|\frac{\rho\omega^2\left(b^{3-k} - a^{3-k}\right)}{2(3-k)b^{1-k}}\right| = Y^*(say)$$

The angular speed required for fully plastic state is given by:

$$\Omega_f^2 = \frac{\rho\omega_f^2 b^2}{Y^*} = \frac{2(3-k)b^{3-k}}{\left(b^{3-k} - a^{3-k}\right)} \tag{27}$$

where $\omega_f = \frac{\Omega_f}{b}\sqrt{\frac{Y}{\rho}}$.

We introduce the following non-dimensional components:

$R = \frac{r}{b}$, $R_0 = \frac{a}{b}$, $\sigma_r = \frac{T_{rr}}{Y}$, $\sigma_\theta = \frac{T_{\theta\theta}}{Y}$, $\bar{u} = \frac{u}{b}$, $\Omega^2 = \frac{\rho\omega^2 b^2}{Y}$ and $H = \frac{Y}{E}$.

Elastic-plastic transitional stresses, angular speed and displacement from equations (22), (23), (26) and (24) in non-dimensional form become:

$$\sigma_\theta = \left(\frac{\Omega_i^2\nu\left(1 - R_0^{3-k}\right)R^{\nu+k-1}}{(3-k)}\right) \tag{28}$$

$$\sigma_r = \frac{\Omega_i^2 R^{k-1}}{(3-k)}\left[\left(1 - R_0^{3-k}\right)R^{\nu} - R^{3-k} + R_0^{3-k}\right] \tag{29}$$

$$\Omega_i^2 = \frac{\rho\omega_i^2 b^2}{Y} = \frac{(3-k)R_0^{1-\nu-k}}{(1-\nu)\left(1 - R_0^{3-k}\right)} \tag{30}$$

$$\bar{u} = R - R\sqrt{1 - 2\nu H \frac{\Omega_i^2}{(3-k)} R^{k-1}\left[R^{3\text{-}k} - {R_0}^{3-k}\right]} \tag{31}$$

Stresses, displacement and angular speed for fully plastic state $(C \to 0)$, are obtained from equations (28), (29),(31) and (27) as:

$$\sigma_\theta = \frac{\Omega_f^2\left(1 - R_0^{3-k}\right)R^k}{2\sqrt{R}(3-k)} \tag{32}$$

$$\sigma_r = \frac{\Omega_f^2 R^{k-1}}{(3-k)}\left[\left(1 - R_0^{3-k}\right)\sqrt{R} - R^{3-k} + R_0^{3-k}\right] \tag{33}$$

$$\bar{u} = R - R\sqrt{1 - H \frac{\Omega_i^2}{(3-k)} R^{k-1}\left[R^{3\text{-}k} - {R_0}^{3-k}\right]} \tag{34}$$

and $\Omega_f^2 = \dfrac{2(3-k)}{\left(1 - R_0^{3-k}\right)}$ (35)

Particular case: When there is $(k = 0)$, the transitional stresses from equations (28) to (31) becomes:

$$\sigma_\theta = \frac{\Omega_i^2 \nu R^{\nu-1}\left(1 - R_0^3\right)}{3} \tag{36}$$

$$\sigma_r = \frac{\Omega_i^2}{3R}\left[\left(1 - R_0^3\right)R^\nu - R^3 + R_0^3\right] \tag{37}$$

$$\Omega_i^2 = \frac{\rho\omega_i^2 b^2}{Y} = \frac{3R_0^{1-\nu}}{(1-\nu)\left(1 - R_0^3\right)} \tag{38}$$

and $\bar{u} = R - R\sqrt{1 - 2\nu H \dfrac{\Omega_i^2}{3R}\left[R^3 - {R_0}^3\right]}$ (39)

For fully plastic state stresses, displacement and angular velocity from equations (36) to (39) become:

$$\sigma_\theta = \frac{\Omega_f^2}{6R} R^{1/2}\left(1 - R_0^3\right) \tag{40}$$

$$\sigma_r = \frac{\Omega_f^2}{3R}\left[\left(1 - R_0^3\right)R^{1/2} - R^3 + R_0^3\right] \tag{41}$$

$$\bar{u} = R - R\sqrt{1 - H \frac{\Omega_f^2}{3R}\left(R^3 - R_0^3\right)} \tag{42}$$

$$\Omega_f^2 = \frac{\rho \omega_f^2 b^2}{Y} = \frac{6}{\left(1 - R_0^3\right)} \tag{43}$$

Equations (36) to (43) are same as given by Pankaj [17].

Table 1. - Angular speed required for initial yielding and fully plastic state

	Variable Thickness k	Compressibility of Material C	Angular Speed required for initial yielding Ω_i^2	Angular Speed required for fully- plastic state Ω_f^2	Percentage increase in Angular speed $\left(\sqrt{\frac{\Omega_f^2}{\Omega_i^2}} - 1\right) \times 100$
$0.5 < R \le 1.0$	-0.5	v =0.5 (Rubber)	3.839354	7.678708	41.42136
	0	(Incompressible	4.848732	6.857143	18.92072
	0.4	Material)	5.810077	6.227086	3.52649
	-0.5	v =0.333	2.563482	7.678708	73.07285
	0	(Copper & Brass)	3.237431	6.857143	45.53633
	0.4	(Compressible	3.879306	6.227086	26.69673
		Material)			
	-0.5	v =0.29	2.33751	7.678708	81.24554
	0	(Steel)	2.952049	6.857143	52.40872
	0.4	(Compressible	3.537343	6.227086	32.67951
		Material			

4. NUMERICAL ILLUSTRATION AND DISCUSSION

For calculating the stresses, angular speed and displacement based on the above analysis, the following values have been taken as v = 0.5 (Rubber or incompressible material), v = 0.333 (Copper and Brass or compressible material), v =0.29 (Steel or compressible material) and k = -0.5, 0, 0.4 respectively. Curves have been drawn in Fig. 2 between angular speed required for initial yielding and various radii ratios for v = 0.5, 0.333, 0.29 at k = -0.5, 0, 0.4. It has been observed that in the absence of thickness the rotating disc made of incompressible material e.g. Rubber with inclusion require higher angular speed to yield at the internal surface as compare to disc made of compressible material e.g. Copper, Brass and Steel and a much higher angular speed is required to yield with the increase in radii ratio. With the effect of variation thickness, higher angular speed is required to yield at the internal surface. It can also be seen from Table- I, that for compressible material higher percentage increased in angular speed is required to become fully plastic as compared to rotating disc made of incompressible material. In Fig. 3(a), 3(b) and 4, curves have been drawn for stresses and displacement with respect to radii ratio R = r/b for elastic-plastic transition and fully plastic state respectively. It has been observed that the radial stress is maximum at the internal surface. With the effect of variable thickness it increases the value of radial and circumferential stress at the internal surface for transitional state whereas from fig.4, it can been seen that rotating disc having variable thickness increases the values of radial and circumferential stress at the internal surface for fully-plastic state.

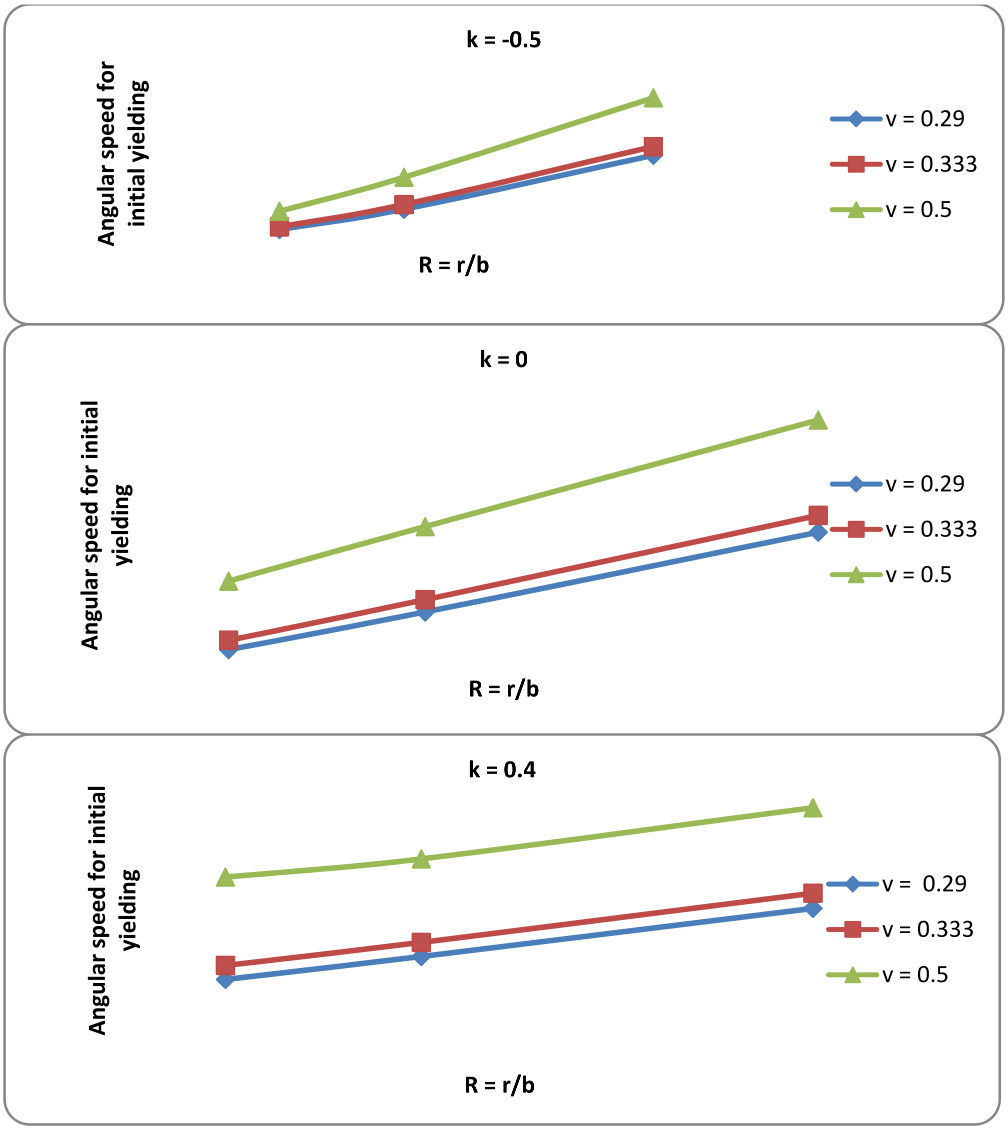

Fig. 2 - Angular speed required for initial yie lding at the internal surface of the rotating disc with rigid inclusion having variable thickness k = -0.5, 0, 0.4 along the radii ratio $R_0 = a/b$

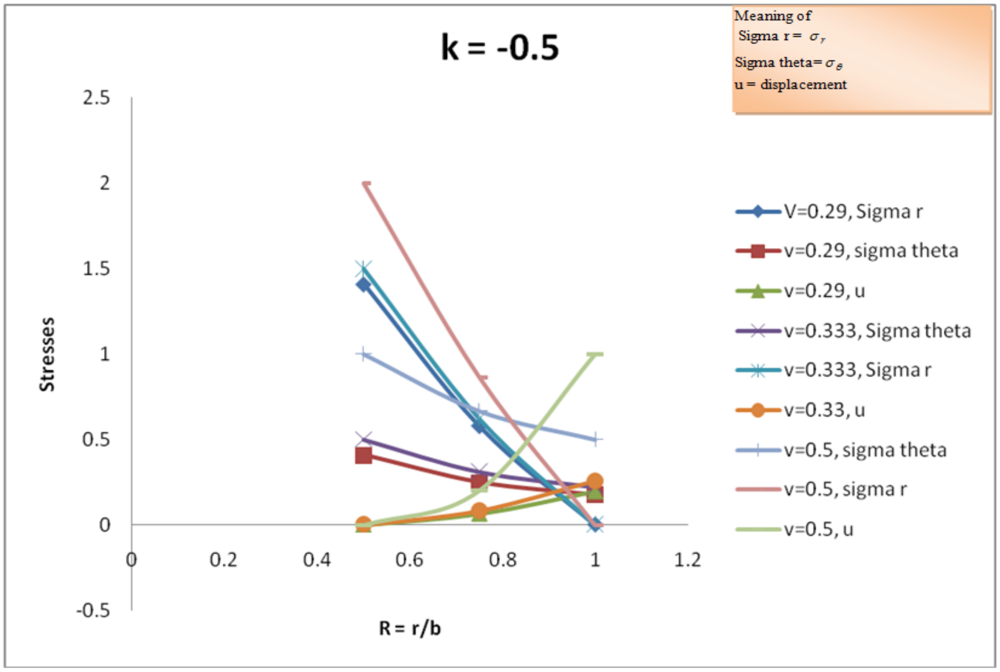

Fig. 3a - Stresses and Displacement at the elastic-plastic Transition state.

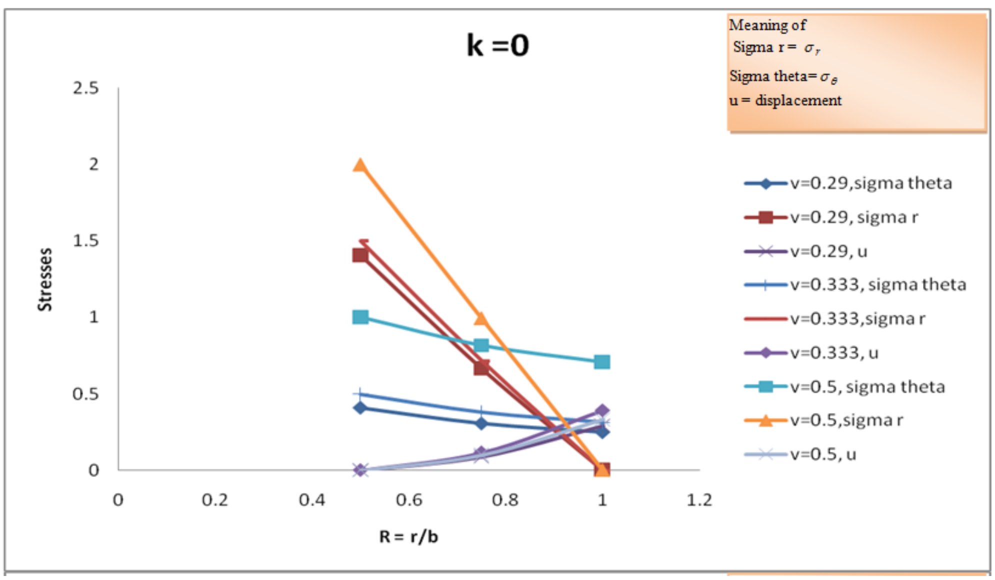

Fig. 3b (1) - Stresses and Displacement at the elastic-plastic Transition state

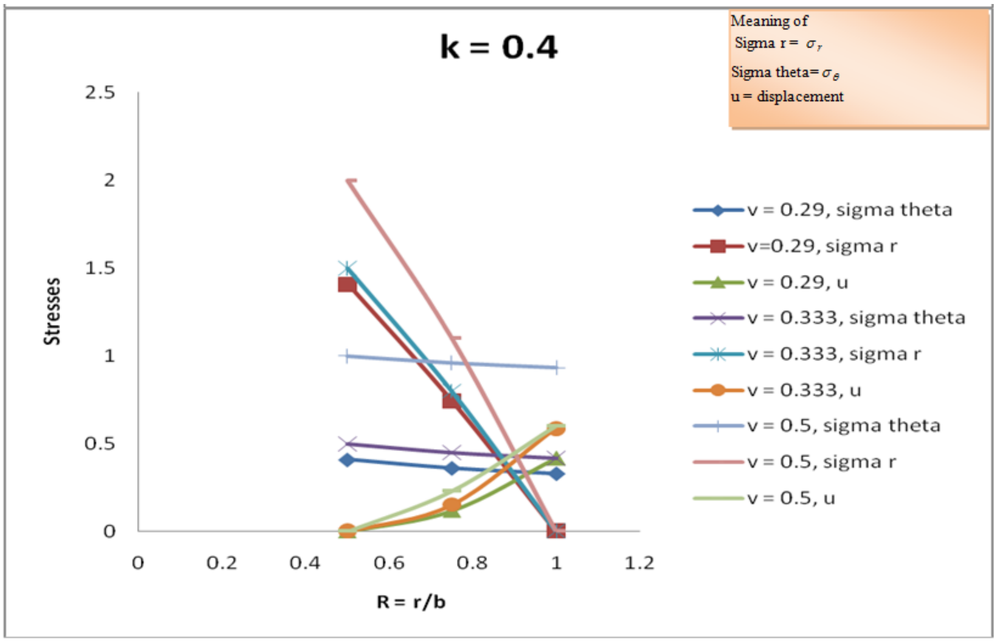

Fig. 3b (2) - Stresses and Displacement at the elastic-plastic Transition state

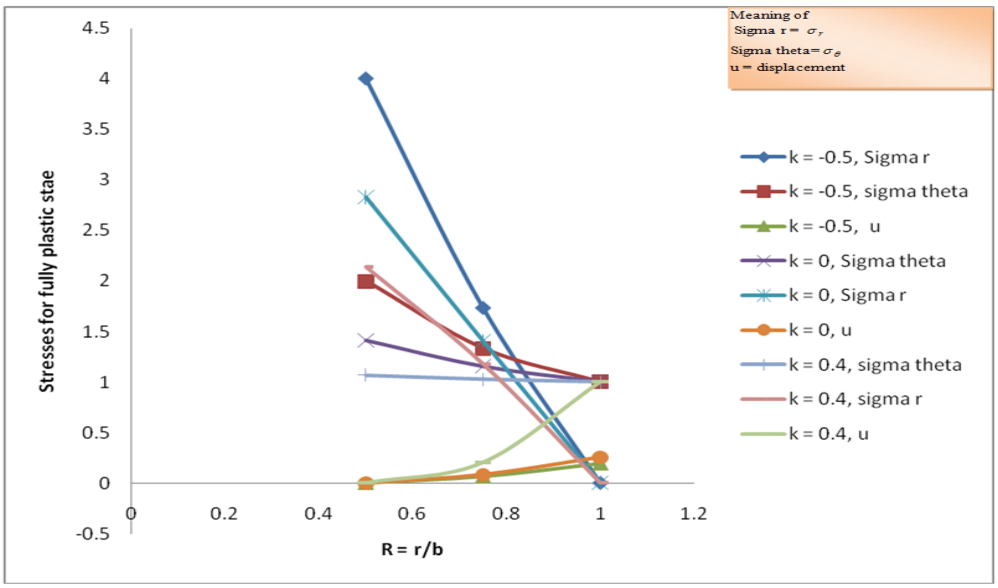

Fig. 4 - Stresses and displacement for fully plastic state

5. CONCLUSION

It has been observed that in the absence of thickness, rotating disc with inclusion and made of compressible material yields at the internal surface at a lesser angular speed as compared to a rotating disc made of incompressible material whereas it requires a higher percentage increase in angular speed to become fully plastic. With the effect of variation thickness, higher angular speed is required to yield at the internal surface. It has been observed that the radial stress is maximal at the internal surface. With the effect of variable thickness it increase the value of radial and circumferential stress at the internal surface for transitional state whereas it can be seen that rotating disc having variable thickness increases the values of radial and circumferential stress at the internal surface for fully-plastic state.

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NAPONI U TANKOM ROTIRAJUĆEM DISKU PROMENLJIVE DEBLJINE SA KRUTIM VRATILOM

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REZIME

Naponi u tankom rotirajućem disku promenljive debljine sa krutim vratilom izvedeni su za elasto-plastično i potpuno plastično stanje pomoću Seth-ove tranzicione teorije. Dobijeni rezultati su diskutovani i grafički prezentovani. Zaključeno je da, kada se zanemari debljina, tečenje materijala (na unutrašnjoj površini) u slučaju rotirajućeg diska izrađenog od stišljivog materijala npr bakra, mesinga i čelika javlja se pri manjoj vrednosti ugaone brzine u odnosu na rotirajući disk napravljen od nestišljivog materijala npr. gume, dok istovremeno zahteva veći procenat povećanja ugaone brzine za prelazak u potpuno plastično stanje. Uključivanjem efekta debljine potrebna je veća ugaona brzina za početak tečenja materijala na unutrašnjoj površini. Maksimalni radijalni napon javlja se na a unutrašnjoj površini. Takođe sa povećanjem debljine povećavaju se vrednost radijalnog i obimnog napona na unutrašnjoj površini za oba naponska tanja (prelno i čisto plastično)

***Ključne reči**: naponi, pomeranja, rotirajući disk, uugaona brzina, inkluzija, debljina*